$\sqrt{2}$ is rational number.

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Official proof.

Euclid's proof starts with the assumption that $\sqrt{2}$ is equal to a rational number $\frac{p}{q}$.

$$\frac{p}{q} = \sqrt{2}$$

Squaring both sides,

$$\frac{p^2}{q^2} = 2$$

The equation can be rewritten as

$$p^2 = 2q^2$$

From this equation, we know p^2 must be even (since it is 2 multiplied by some number). Since p^2 is an even number, it can be inferred that p is also an even number.
Since p is even, it can be written as 2m where m is some other whole number. This is because the definition of an even number is it can be written as 2 multiplied by a whole number. Substituting p=2m in the above equation:

$$2q^2 = 4m^2$$

Dividing both sides of the equation by 2:

$$q^2 = 2m^2$$

By the same reasoning as before, q^2 is an even number (since it is written as 2 multiplied by some number). So q is an even number. Let q=2n where n is some whole number. We had assumed $\sqrt{2}$ to be equal to p/q. So:

$$\frac{p}{q} = \frac{2m}{2n} = \frac{m}{n} = \sqrt{2}$$

We now have a fraction m/n simpler than p/q. However, we now find ourselves in a position whereby we can repeat exactly the same process on m/n, and at the end of it, we can generate a simpler one, say g/h. This fraction can be put through the same process again, and the new fraction, say, e/f will be simpler again. But we know that rational number cannot be simplified indefinitely. There must always be a simplest rational number and the original assumption that $\sqrt{2}$ is equal to p/q does not obey this rule. So it can be stated that a contradiction has been reached. If $\sqrt{2}$ could be written as a rational number, the consequence would be absurd. So it is true to say that $\sqrt{2}$ cannot be written in the form p/q. Hence $\sqrt{2}$ is not a rational number. Thus, Euclid succeeded in proving that $\sqrt{2}$ is an Irrational number.

What's wrong with canonical proof?

In fact, definition of the odd/even has absolutely no sense for rational numbers, thereby we must take 2 in the form suitable fo rationals:

$$\frac{p^2}{q^2} = 1.\underbrace{99..99999999}_{Infinite}$$

So now we can easily compute:

$$\frac{1414^2}{1000^2} = 1,999396$$

Or

$$\frac{1414213415^2}{1000000011^2} = 1,999999539165972$$