

## Factorization nr.888

Part I

If  $N=4*G+3 \rightarrow N1=N ; N2=5*N$

If  $N=4*G+1 \rightarrow N1=3*N ; N2=3*5*N$

If  $(N1-3) \bmod 8 == 0 \rightarrow N=N1$

else if  $(N2-3) \bmod 8 == 0 \rightarrow N=N2$

$$(N-3)/8-q*(p-A)/8-[4-(A-7)*(A-5)/8]=A*(q+A-4-8)/8$$

$\rightarrow$

$$N=p*q$$

if we choose A such that  $p-A \bmod 8 = 0$  and we choose A of the same size order as q with  $A < q$

we can write it this way

$$(N-3)/8-Q-[4-(A-7)*(A-5)/8]=A*X$$

so there are 4 chances to find A and they are

$$8*h+1 ; 8*h+3 ; 8*h+5 ; 8*h+7$$

$$(N-3)/8-p*(q-B)/8-[4-(B-7)*(B-5)/8]=B*(p+B-4-8)/8$$

$\rightarrow$

$$N=p*q$$

if we choose B such that  $q-B \bmod 8 = 0$  and we choose B of the same size order as p with  $B < p$

we can write it this way

$$(N-3)/8-P-[4-(B-7)*(B-5)/8]=B*Y$$

so there are 4 chances to find B and they are

$$8^k+1 ; 8^k+3 ; 8^k+5 ; 8^k+7$$

$f(N,A,B)$  is  $O(16)$

Example

$$1763=41*43$$

$$220-q*(p-25)/8-[4-(25-7)*(25-5)/8]=25*(q+25-4-8)/8$$

$$220-p*(q-27)/8-[4-(27-7)*(27-5)/8]=27*(p+27-4-8)/8$$

$$220-Q-[4-(25-7)*(25-5)/8]=25*X$$

$$220-P-[4-(27-7)*(27-5)/8]=27*X$$

$$Q=25*y+11 \quad X=10-y$$

$$P=27*x+1 \quad X=10-x$$

$$\text{solve } Q=q*(p-25)/8=25*y+11, p*q=1763$$

$$\rightarrow q=67-8*y$$

$$\text{solve } p*(q-27)/8=27*x+1, p*q=1763$$

$$\rightarrow p=65-8*x$$

## Part II Factorization example

The whole is based on a partial factorization of another number (which we can build ourselves (in the example  $451328=8*g$ ))

$$P=27*x+1 ; p=65-8*x$$

$$Q=25*y+11 ; q=67-8*y$$

$$K=(65-8*x)^2=64*x^2-1040*x+4225$$

$$(27*h-1040)^2-4*64*(4225+h)=D^2$$

$$h=1/729*(28208-\text{sqrt}(729*D^2+795691264))$$

$$795691264=1763*451328$$

$$\text{sqrt}[729*D^2+795691264]-\text{sqrt}[729*D^2]=8*(67-8*y)$$

replace D

$$\text{sqrt}[729*((27*h-1040)^2-4*64*(4225+h))+795691264]-27*\text{sqrt}[(27*h-1040)^2-4*64*(4225+h)]=8*(67-8*y)$$

$$28208-729*h-27*\text{sqrt}[(27*h-1040)^2-4*64*(4225+h)]=8*(67-8*y)$$

$$\rightarrow h=4*(64*y^2+55344*y+11964681)/(729*(8*y-67))$$

$$64*y^2+55344*y+11964681=729*W$$

$$55344+729*a=64*b \rightarrow a=16$$

$$11964681+729*c=64*d \rightarrow c=15$$

$$[64/64*y^2+((55344+16*729)/64 \bmod 729)*y+((11964681+15*729)/64 \bmod 729)] \bmod 729 = 0$$

$$y^2+318*y+495 \bmod 729 = 0$$

apply coppersmith method

$$\rightarrow y=3 \rightarrow q=43$$

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