Proof of Beal's Conjecture and Fermat's Last Theorem

Fermat's Last Theorem

$P^n + Q^n \neq R^n$, $n > 2$ if P, Q, R and n are positive integers.
Andrew Wiles produced a lengthy proof of over 100 pages in the mid 1990s.

Beal's Conjecture

If $A^x + B^y = C^z$, $x > 2$, $y > 2$, $z > 2$, where A, B, C, x, y and z are positive integers, then A, B and C must have a highest common factor > 1.

If n > 2 is an odd integer, the left hand side of the equation can be written as
$A^x + B^y = (A^{x/n})^n + (B^{y/n})^n = [A^{x/n} + B^{y/n}][(A^{x/n})^{n-1} - (A^{x/n})^{n-2} B^{y/n} + (A^{x/n})^{n-3} (B^{y/n})^2 - \ldots + (A^{x/n})^2 (B^{y/n})^{n-3} - A^{x/n}(B^{y/n})^{n-2} + (B^{y/n})^{n-1}]$. Beal's conjecture equation can be written as $C^z - B^y = A^x$. If n > 2 is an integer, the left hand side of the equation can be written as
$C^z - B^y = (C^{z/n})^n - (B^{y/n})^n = [C^{z/n} - B^{y/n}][(C^{z/n})^{n-1} + (C^{z/n})^{n-2} B^{y/n} + (C^{z/n})^{n-3} (B^{y/n})^2 + \ldots + (C^{z/n})^2 (B^{y/n})^{n-3} + C^{z/n}(B^{y/n})^{n-2} + (B^{y/n})^{n-1}]$

Proof of Beal's Conjecture

Let it be initially assumed that A and B have a highest common factor = 1 and $A^x + B^y = (A^{x/n})^n + (B^{y/n})^n = [A^{x/n} + B^{y/n}][(A^{x/n})^{n-1} - (A^{x/n})^{n-2} B^{y/n} + (A^{x/n})^{n-3} (B^{y/n})^2 - \ldots + (A^{x/n})^2 (B^{y/n})^{n-3} - A^{x/n}(B^{y/n})^{n-2} + (B^{y/n})^{n-1}] = M^n$ where n > 2 is an odd integer. Let $M = d - e$, $A^{x/n} = hd$, $B^{y/n} = ie$ where M is a positive integer, d and e are positive rational numbers.
Therefore $[A^{x/n} + B^{y/n}][(A^{x/n})^{n-1} - (A^{x/n})^{n-2} B^{y/n} + (A^{x/n})^{n-3}(B^{y/n})^2 - \ldots + (A^{x/n})^2(B^{y/n})^{n-3} - A^{x/n}(B^{y/n})^{n-2} + (B^{y/n})^{n-1}]/M = M^{n-1}$ i.e.
$[(hd + ie)/(d - e)]h^{n-1}d^{n-1} - [(hd + ie)/(d - e)]h^{n-2}\, i\, d^{n-2} e$
$+[(hd + ie)/(d - e)]h^{n-3} i^2 d^{n-3} e^2 - \ldots +[(hd + ie)/(d - e)]h^2 i^{n-3} d^2 e^{n-3}$
$- [(hd + ie)/(d - e)]h\, i^{n-2} d\, e^{n-2} +[(hd + ie)/(d - e)]\, i^{n-1} e^{n-1}$
$= d^{n-1} - (n-1)d^{n-2} e + \binom{n-1}{2} d^{n-3} e^2 - \ldots + \binom{n-1}{2} d^2 e^{n-3} - (n-1)\, d\, e^{n-2} + e^{n-1}$
Equating the first and last coefficients on the left hand side of the equation to the first and last binomial coefficient of 1 on the right hand side of the equation implies that h = i. Equating the second and second to the last coefficients on the left hand side of the equation to

the second and second to the last binomial coefficient of n-1 on the right hand side of the equation implies that h = i once again. If this is carries on, h = i every time because of the symmetry of the binomial coefficients. If h = i, this implies that all coefficients on the left hand side of the equation are the same. If corresponding coefficients on both sides of the equation are equated given that h = i then conflicting values of h = i begin to arise. The left hand side of the equation can therefore not be equal to the right hand side of the equation if A and B have a highest common factor = 1. Therefore $A^x + B^y \neq M^n$ if A and B have a highest common factor =1. This proves Beal's conjecture, $A^x + B^y \neq C^z$ when z is an odd integer, A and B have a highest common factor = 1.

Let it be initially assumed that C and B have a highest common factor = 1 and $C^z - B^y = (C^{z/n})^n - (B^{y/n})^n = [C^{z/n} - B^{y/n}][(C^{z/n})^{n-1} + (C^{z/n})^{n-2} B^{y/n} + (C^{z/n})^{n-3} (B^{y/n})^2 + ... + (C^{z/n})^2 (B^{y/n})^{n-3} + C^{z/n}(B^{y/n})^{n-2} + (B^{y/n})^{n-1}] = L^n$. Let L = f + g, $B^{y/n} = jf$ and $C^{z/n} = kg$ where L is a positive integer, f and g are positive rational numbers. Therefore $[C^{z/n} - B^{y/n}][(C^{z/n})^{n-1} + (C^{z/n})^{n-2} B^{y/n} + (C^{z/n})^{n-3} (B^{y/n})^2 + ... + (C^{z/n})^2 (B^{y/n})^{n-3} + C^{z/n}(B^{y/n})^{n-2} + (B^{y/n})^{n-1}]/L = L^{n-1}$ i.e.
$[(kg – jf)/(f + g)]k^{n-1} g^{n-1} +[(kg – jf)/(f + g)]k^{n-2} j g^{n-2} f +[(kg – jf)/(f + g)]k^{n-3} j^2 g^{n-3} f^2 + …+[(kg – jf)/(f + g)]k^2 j^{n-3} g^2 f^{n-3} +[(kg – jf)/(f + g)]k j^{n-2} g f^{n-2} +[(kg – jf)/(f + g)] j^{n-1} f^{n-1} = g^{n-1} + (n-1)g^{n-2} f + \binom{n-1}{2} g^{n-3} f^2 + … + \binom{n-1}{2} g^2 f^{n-3} + (n-1) g f^{n-2} + f^{n-1}$
Equating the first and last coefficients on the left hand side of the equation to the first and last binomial coefficient of 1 on the right hand side of the equation implies that j = k. Equating the second and second to the last coefficients on the left hand side of the equation to the second and second to the last binomial coefficient of n-1 on the right hand side of the equation implies that j = k once again. If this is carries on, j = k every time because of the symmetry of the binomial coefficients. If j = k, this implies that all coefficients on the left hand side of the equation are the same. If corresponding coefficients on both sides of the equation are equated given that j = k then conflicting values of j = k begin to arise. The left hand side of the equation can therefore not be equal to the right hand side of the equation if C and B have a highest common factor = 1. Therefore $C^z - B^y \neq L^n$ if C and B

have a highest common factor = 1. This proves Beal's conjecture, $C^z - B^y \neq A^x$ if C and B have a highest common factor = 1. It should now be clear why all examples of Beal's conjecture that have x, y and z to be different all originate from Beal conjecture examples where 2 of x, y or z are the same e.g. $7^6 + 7^7 = 98^3$ can be rewritten as $49^3 + 7^7 = 98^3$. $242^{10} + 242^{11} = 175692^5$ can be rewritten as $58564^5 + 242^{11} = 175692^5$ or $242^5 + 242^6 = 726^5$ if each of the terms is divided by 242^5.

Proof of Fermat's Last Theorem

It has just been shown that $(C^{z/n})^n - (B^{y/n})^n \neq L^n$. Let $C^{z/n}$ and $B^{y/n}$ be integers. Let $tC^{z/n} = R$, $tB^{y/n} = Q$ and $tL = P$, where t is the highest common factor of P, Q and R. When each term in the inequality $(C^{z/n})^n - (B^{y/n})^n \neq L^n$ is multiplied by t^n then $R^n - Q^n \neq P^n$ i.e. Fermat's Last theorem is proved. For the special case when n = 2 i.e. the Pythagorean theorem, there will be no conflicting values of j and k. Therefore, the condition of A, B and C sharing a highest common factor > 1 is not necessary.