Search for Prime Constellations

Prime Constellations are more constant than the stars in the sky.

Even after the closest stars burn out and become dark, mathematical structure will exist. For example 1+1=2 is a
timeless mathematical truth. This fact will always be so.

Look at this link about Goldbach's Comet.

Also interesting Tao-Green Theorem

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A table of the first few prime numbers contains data that has a certain mathematical property. Similarly, a table
twin primes also contains some mathematical truth. To me, it is interesting to develop these tables. The Univers
Tennessee at Martin currently has two web pages for primes and twin primes.

specifically
http://primes.utm.edu/lists/small/1000.txt
and
http://primes.utm.edu/lists/small/100ktwins.txt

Table of contents

Background

Prime numbers are positive integers that are divisible only by one and themselves. The first few prime numbers
3, 5, 7, 11, 13 and 17. There are many unsolved problems regarding prime numbers. There may be a hidden patt
that describes the distribution of the primes. Goldbach's conjecture states that every even integer greater than 2 is
sum of two prime numbers. It has not been shown to be true for certain, although the numerical evidence suppor
The Hardy Littlewood second conjecture states that the densest set of n prime numbers is the first n prime numbe
This can be written in symbols

$\pi(x+n)$ - $\pi(x)$ is less than or equal to $\pi(n)$

$\pi(n)$ is the prime counting function.

The wikipedia page on prime k-tuples is a good introduction. Additionally, Tony Forbes keeps a record of primes in various constellation patterns, Jens Krus Anderson keeps a nice webpage about prime numbers at http://primerecords.dk/ and Wolfram's Mathw has a nice article about prime constellations. Each constellation of a given length has a fixed number of patterns. The list of number o patterns for a given k-tuplet can be found in the Online Encyclopedia of Integer Sequences (OEIS), index A083409.

The k-tuple conjecture is also called the first Hardy and Littlewood conjecture. Being a conjecture, it is an unpro guess. It states that every admissable pattern gives rise to an infinite number of primes and the asymptotic densit theses primes can be calculated. An admissible pattern is usually written with the first number as zero. Then the pattern is admissible if it does not contain a trivial divisibility that prevents an infinite set of primes that fit into t pattern. For example, any admissable pattern that starts with 0 cannot contain an odd number. Because there is one even prime and all the rest are odd. Similarly, three consecutive odd numbers is not an admissable set becau one of divisbility by 3 considerations. To check a set of length k, one must test all the entries in the potentialy admissable set against the primes less than k to makes sure all the residue classes are not filled up. There is a nic computer program that will test a set for admissability, and it can be found here.

Encyclopedia Tables

Authors -

N.J.A. Slone = NJAS
Warut Roonguthai = WR
Matt C. Anderson = MCA
Tim Johannes Ohrtmann = TJO

The Online Encyclopedia of Integer Sequences has prime lists for several patterns:
A001359 Initial member of twin primes pattern (0,2) enumeration count 100,000 author NJAS
A022004 Initial member of prime triple (0,2,6) enumeration count 10,000 author WR
A022005 Initial member of prime triple (0,4,6) enumeration count 10,000 author WR
A007530 Initial member of prime quadruple (0,2,6,8) enumeration count 10,000 author WR
A022006 Initial member of prime 5-tuplet (0,2,6,8,12) enumeration count 10,000 author WR
A022007 Initial member of prime 5-tuplet (0,4,6,10,12) enumeration count 10,000 author WR
A022008 Initial member of prime 6-tuplet (0,4,6,10,12,16) enumeration count 1,000 author WR
A022009 Initial member of prime 7-tuplet (0,2,6,8,12,18,20) enumeration count 10,000 author MCA
A022010 Initial member of prime 7-tuplet (0,2,8,12,14,18,20) enumeration count 10,000 author MCA
A022011 Initial member of prime 8-tuplet (0,2,6,8,12,18,20,26) enumeration count 10,000 author WR
A022012 Initial member of prime 8-tuplet (0,2,6,12,14,20,24,26) enumeration count 10,000 author WR
A022013 Initial member of prime 8-tuplet (0,6,8,14,18,20,24,26) enumeration count 10,000 author WR
A022545 Initial member of prime 9-tuplet (0,2,6,8,12,18,20,26,30) enumeration count 10,000 author WR
A022546 Initial member of prime 9-tuplet (0,2,6,12,14,20,24,26,30) enumeration count 10,000 author MCA
A022547 Initial member of prime 9-tuplet (0,4,6,10,16,18,24,28,30) enumeration count 10,000 author WR
A022548 Initial member of prime 9-tuplet (0,4,10,12,18,22,24,28,30) enumeration count 10,000 author WR
A027569 Initial member of prime 10-tuplet (0,2,6,8,12,18,20,26,30,32) enumeration count 10,000 author WR
A027570 Initial member of prime 10-tuplet (0,2,6,12,14,20,24,26,30,32) enumeration count 10,000 author WR
A213646 Initial member of prime 11-tuplet (0,4,6,10,16,18,24,28,30,34,36) enumeration count 6923 author MCA

A213647 Initial member of prime 11-tuplet (0,2,6,8,12,18,20,26,30,32,36) enumeration count 6800 author MCA
A213645 Initial member of prime 12-tuplet (0,2,6,8,12,18,20,26,30,32,36,42) enumeration count 2807 author MCA
A213601 Initial member of prime 12-tuplet (0,6,10,12,16,22,24,30,34,36,40,42) enumeration count 2952 author
A234947 Initial members of prime 13-tuplet (0, 6, 12, 16, 18, 22, 28, 30, 36, 40, 42, 46, 48) enumeration count 854 author MCA

D. Jacobson has written some software that will calculate even longer lists of prime constellations which are a su of the k-tuples.

A257137 Initial members of prime 13-tuplet (0, 4, 6, 10, 16, 18, 24, 28, 30, 34, 40, 46, 48) enumeration count 94 TJO
A257138 Initial members of prime 13-tuplet (0, 4, 6, 10, 16, 18, 24, 28, 30, 34, 36, 46, 48) enumeration count 80 TJO
A257139 Initial members of prime 13-tuplet (0, 2, 6, 8, 12, 18, 20, 26, 30, 32, 36, 42, 48) enumeration count 81
A257140 Initial members of prime 13-tuplet (0, 2, 8, 14, 18, 20, 24, 30, 32, 38, 42, 44, 48) enumeration count 1036 author TJO
A257141 Initial members of prime 13-tuplet (0, 2, 12, 14, 18, 20, 24, 30, 32, 38, 42, 44, 48) enumeration count 803 author TJO

Tim Johannes Ohrtmann posted some primes for the OEIS for 14 to 17-tuplets.

A257167 Initial members of prime 14-tuplet (0, 2, 6, 8, 12, 18, 20, 26, 30, 32, 36, 42, 48 and 50) enumeration c 185 author TJO
A257168 Initial members of prime 14-tuplet (0, 2, 8, 14, 18, 20, 24, 30, 32, 38, 42, 44, 48 and 50) enumeration 209 author TJO
A257304 Initial members of prime 15-tuplet (0, 2, 6, 8, 12, 18, 20, 26, 30, 32, 36, 42, 48, 50 and 56) enumeratio count 16 author TJO

A257304 only has 8 calculated primes as of 9/22/2015.

A257369 Initial members of prime 16-tuplet (0,4,6,10,16,18,24,28,30,34,40,46,48,54,58, 60) enumeration coun 30 author TJO
A257370 Initial members of prime 16-tuplet (0,2,6,12,14,20,26,30,32,36,42,44,50,54,56,60) enumeration cou 37 author TJO
A257374 Initial members of prime 17-tuplet (0,4, 10,12, 16, 22, 24, 30, 36, 40, 42, 46, 52, 54, 60, 64, 66) enumeration count 2 author TJO
A257375 Initial members of prime 17-tuplet (0,4,6,10,16,18,24,28,30,34,40,46,48,54,58,60,66) enumeration c 6 author TJO
A257376 Initial members of prime 17-tuplet (0,6,8,12,18,20,26,32,36,38,42,48,50,56,60,62,66) enumeration c 4 author TJO
A257377 Initial members of prime 17-tuplet (0,2,6,12,14,20,24,26,30,36,42,44,50,54,56,62,66) enumeration c 8 author TJO

Tony Forbes has a nice website with prime constellations and world record large k-tuplets.
http://anthony.d.forbes.googlepages.com/ktmin.txt
and
http://anthony.d.forbes.googlepages.com/ktuplets.htm

note - A213601 has a list of 73 primes and represents over 8 months of Maple computer calculation as of 8/23/20

A008407 gives the minimum width of a constellation of given length.

A257127 is Initial members of prime 10-tuplets (or decaplets) and lots of cross-references.

Theory

Two concepts are important for the theory of prime constellations. The first is the concept of *primorial*.

k primorial (written k#) is the product of the first k primes. Let pk be the kth prime

k	pk	k#
1	2	2
2	3	6
3	5	30
4	7	210

The numbers in the third column are the product of the primes in the second column.

The second concept is part of group theory. The multiplicative group of integers modulo n must be understood. is the wikipedia article about this multiplicative group. The notation reads (Z/nZ)*. It contains a subset of the integers from 1 to n-1. The elements of (Z/nZ)* are the integers from 1 to n-1 that are relatively prime to n. If n prime number, then (Z/nZ)* contains all the integers from 1 to n-1. If n has many divisors, then (Z/nZ)* will co fewer elements.

One way to find these prime k-tuplets is to consider the multiplicative group of integers mod k primorial. This g contains the set of integers less than k primorial that are relatively prime to k primorial.

2# = 6

The multiplicative group mod 6 has two elements.
(Z/6Z)* = {1,5}
This tells us that all primes greater than 3 have the form 6n +/- 1.
So if one wants to search for the smaller of twin prime pairs, one should look at numbers of the form 6*n+5.
It would be a waste of time to test even numbers for primality, because all primes greater than 2 are odd.

Similarly, the group mod 30 has 8 elements.
(Z/30Z)* = {1,7,11,13,17,19,23,29}
By looking at the differences between adjacent elements in this set, we can see where the 3 tuplets can be found. prime triples of the form (p,p+2,p+6) can be found only in the expressions 30*n+11 and 30*n+17.

Also, constellations of length 4, which are similarly called 4-tuplets, have the pattern (p, p+2,p+6,p+8).
Close inspection of the set {1,7,11,13,17,19,23,29} gives that the 4-tuplets must have the form 30*n+11. So the step is to loop with the variable n and test for all the elements in the pattern being prime.

The ordered set (Z/30Z)* = {1,7,11,13,17,19,23,29} can be manipulated by taking the differences between adjac elements.

d30 = [6,4,2,4,2,4,6]
This shows that the pattern (p,p+2,p+6,p+8), which has differences [2,4,2], can be found inside the ordered set d

The file below ktpatt.txt was copied from Anthony Forbe's website, and shows the patterns for constellations up length 50.

Data for finding Constellations

tuple	pattern	offset and multiplier
2	(0,2)	5+6*n
3	(0,2,6)	5+6*n
3	(0,4,6)	1+6*n
4	(0,2,6,8)	11+30*n
5	(0,2,6,8,12)	11+30*n
5	(0,4,6,10,12)	7+30*n
6	(0,4,6,10,12,16)	97+210*n
7	(0,2,6,8,12,18,20)	11+210*n
7	(0,2,8,12,14,18,20)	179+210*n
8	(0,2,6,8,12,18,20,26)	11+210*n

8	(0,6,12,14,20,24,26)	17+30*n
8	(0,6,8,14,18,20,24,26)	173+210*n
9	(0,2,6,8,12,18,20,26,30)	11+210*n
9	(0,2,6,12,14,20,24,26,30)	17+30*n
9	(0,4,6,10,16,18,24,28,30)	13+30*n
9	(0,4,10,12,18,22,24,28,30)	169+210*n
10	(0,2,6,8,12,18,20,26,30,32)	11+210*n
10	(0,2,6,12,14,20,24,26,30,32)	167+210*n
11	(0,4,6,10,16,18,24,28,30,34,36)	1003+2310*n
11	(0,2,6,8,12,18,20,26,30,32,36)	1271+2310*n
12	(0,2,6,8,12,18,20,26,30,32,36,42)	997+2310*n
12	(0,6,10,12,16,22,24,30,34,36,40,42)	1271+2310*n

length	pattern	relation
13	0 6 12 16 18 22 28 30 36 40 42 46 48	991 + 2310 * n
13	0 4 6 10 16 18 24 28 30 34 40 46 48	13 + 210 * n

13 0 4 6 10 16 18 24 28 30 34 36 46 48	1003 + 2310 * n
13 0 2 6 8 12 18 20 26 30 32 36 42 48	1271 + 2310 * n
13 0 2 8 14 18 20 24 30 32 38 42 44 48	149 + 210 * n
13 0 2 12 14 18 20 24 30 32 38 42 44 48	1259 + 2320 * n
14 0 2 6 8 12 18 20 26 30 32 36 42 48 50	15131 + 30030 * n
14 0 2 8 14 18 20 24 30 32 38 42 44 48 50	14849 + 30030 * n
15 0 2 6 8 12 18 20 26 30 32 36 42 48 50 56	15141 + 30030 * n
15 0 2 6 12 14 20 24 26 30 36 42 44 50 54 56	17 + 210 * n
15 0 2 6 12 14 20 26 30 32 36 42 44 50 54 56	137 + 210 * n
15 0 6 8 14 20 24 26 30 36 38 44 48 50 54 56	14843 + 30030 * n
16 0 4 6 10 16 18 24 28 30 34 40 46 48 54 58 60	6943 + 30030 * n
16 0 2 6 12 14 20 26 30 32 36 42 44 50 54 56 60	23027 + 30030 * n
17 0 4 10 12 16 22 24 30 36 40 42 46 52 54 60 64 66	2227 + 2310 * n
17 0 4 6 10 16 18 24 28 30 34 40 46 48 54 58 60 66	6943 + 30030 * n
17 0 6 8 12 18 20 26 32 36 38 42 48 50 56 60 62 66	23021 + 30030 * n
17 0 2 6 12 14 20 24 26 30 36 42 44 50 54 56 62 66	17 + 2310 * n
18 0 4 10 12 16 22 24 30 36 40 42 46 52 54 60 64 66 70	23017 + 30030 * n
18 0 4 6 10 16 18 24 28 30 34 40 46 48 54 58 60 66 70	6943 + 30030 * n
19 0 6 10 16 18 22 28 30 36 42 46	293281 +

	48 52 58 60 66 70 72 76	510510 * n
19	0 4 6 10 16 22 24 30 34 36 42 46 52 60 64 66 70 72 76	37 + 30030 * n
19	0 4 6 10 12 16 24 30 34 40 42 46 52 54 60 66 70 72 76	29917 + 30030 * n
19	0 4 6 10 16 18 24 28 30 34 40 46 48 54 58 60 66 70 76	217153 + 510510 * n
20	0 2 6 8 12 20 26 30 36 38 42 48 50 56 62 66 68 72 78 80	29921 + 30030 * n
20	0 2 8 12 14 18 24 30 32 38 42 44 50 54 60 68 72 74 78 80	29 + 30030 * n
21	0 4 6 10 12 16 24 30 34 40 42 46 52 54 60 66 70 72 76 82 84	29917 + 30030 * n
21	0 2 8 12 14 18 24 30 32 38 42 44 50 54 60 68 72 74 78 80 84	29 + 30030 * n
22	0 4 6 10 12 16 24 30 34 40 42 46 52 54 60 66 70 72 76 82 84 90	510397 + 510510 * n
22	0 4 10 12 18 22 24 28 34 40 42 48 52 54 60 64 70 78 82 84 88 90	19 + 510510 * n
22	0 2 6 8 12 20 26 30 36 38 42 48 50 56 62 66 68 72 78 80 86 90	510401 + 510510 * n
22	0 6 8 14 18 20 24 30 36 38 44 48 50 56 60 66 74 78 80 84 86 90	23 + 510510 * n
23	0 4 6 10 12 16 24 30 34 40 42 46 52 54 60 66 70 72 76 82 84 90 94	510397 + 510510 * n
23	0 4 10 12 18 22 24 28 34 40 42 48 52 54 60 64 70 78 82 84 88 90 94	19 + 510510 * n

length	pattern	offset and multiplier
24	0, 4, 6, 10, 12, 16, 24, 30, 34, 40, 42, 46, 52, 60, 66, 70, 72, 76, 82, 84, 90, 94, 96, 100	108457+510510*n
24	0, 4, 6, 12, 16, 24, 30, 34, 40, 42, 46, 52, 54, 60, 66, 70, 72, 76, 82, 84, 90, 94, 96, 100	293257+510510*n
24	0, 4, 6, 10, 16, 18, 24, 28, 30, 34, 40, 46, 48, 54, 58, 60, 66, 70, 76, 84, 88, 94, 96, 100	217513+510510*n
24	0, 4, 6, 10, 16, 18, 24, 28, 30, 34, 40, 48, 54, 58, 60, 66, 70, 76, 84, 88, 90, 94, 96, 100	401953+510510*n

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Also, see -
Our page on 2-tuples
http://www.fermatsearch.org/
Although the search for factors of large numbers is not as good as the Folding__@__Home project by Stanford University, which could save a life, it is still fun.
(worth a look)
try the Google words
(search)
folding at home
and here is the link
https://folding.stanford.edu/
This project is more than 16 years old.

References

Weisstein, Eric W. "k-Tuple Conjecture." From *MathWorld*--A Wolfram Web Resource. http://mathworld.wolfram.com/k-TupleConjecture.html

Caldwel, Chris K "Prime k-tuple conjecture" From The Prime Glossary http://primes.utm.edu/glossary/xpage/PrimeKtupleConjecture.html