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(Greetings from The On-Line Encyclopedia of Integer Sequences!)

Search: **matt c anderson**

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A201998 Positive numbers n such that n^2 + n + 41 is composite and there are no positive that n = **c***x^2 + (**c** + 1)*x + **c***41 for an integer x.

244, 249, 251, 266, 270, 295, 301, 336, 344, 389, 399, 407, 416, 418, 445, 449, 454, 466, 489, 494, 496, 500, 506, 527, 531, 545, 547, 563, 570, 571, 5

620, 622, 624, 628, 630, 636, 652, 661, 662, 663, 679, 693, 699 (list; graph; refs; listen; history; edit; text; internal format)

OFFSET 1,1

COMMENTS The composition of functions k(x) factors. k(x) = (x^2 + x + 41)*(**c**^2*x^2 + (**c**^2 + 2***c**)*x + **c**^2*41 + **c** + 1). So k(x) is the p
greater than one and thus composite.

REFERENCES John Stillwell, Elements of Number Theory, Springer, 2003, page 3.

LINKS Table of n, a(n) for n=1..48.
Matt C. Anderson A prime producing polynomial writeup

MAPLE

```
maxn:=1000:
A:={}:
for n from 1 to maxn do
g:=n^2+n+41:
if isprime(g)=false then
A:=A union {n}:
end if:
end do:
# The set A contains values n such that n^2+n+41 is composite and n < maxn.
c:=1:
x:=-1:
p:=41:
q:=c*x^2-(c+1)*x+c*p:
A2:=A:
while q < maxn do
while q < maxn do
A2:=A2 minus {q}:
A2:=A2 minus {c*x^2+(c+1)*x+c*p}:
x:=x+1:
q:=c*x^2-(c+1)*x+c*p:
end do:
c:=c+1:
x:=-1:
q:=c*x^2-(c+1)*x+c*p:
end do:
A2;
```

MATHEMATICA

```
Reap[For[n=1, n<700, n++, If[!PrimeQ[n^2+n+41], If[Reduce[c>0 && n == c*x^2+(c+1)*x+41*c , {c, x}, Integers] === False, Sow[n]
    François Alcover, Apr 30 2014 *)
```

CROSSREFS Cf. A007634 (n^2 + n + 41 is composite).
Cf. A235381 (similar to this sequence).

KEYWORD nonn

AUTHOR **_Matt C. Anderson_**, Dec 07 2011

STATUS approved

A235381 Positive numbers n such that n^2 + n + 41 is composite and there are no positive such that n = **c***d*x^2 + ((d-2)***c** + 1)*x + ((41*d^2 - d + 1)***c** -1)/d for an intege

611, 622, 630, 663, 679, 734, 758, 835, 867, 966, 978, 995, 1006, 1009, 1060, 1088, 1127, 1142, 1157, 1173, 1175, 1183, 1228, 1280, 1345, 1355, 1368, 1388, 1390, 1426, 1433, 1455, 1457, 1467, 1497, 1538, 1539, 1543, 1554, 1578, 1603, 1612, 1613, 1630, 1661 (list; graph; refs; listen; history; edit; text; internal format)

OFFSET 1,1

COMMENTS Restricting **c** and d so that **c** is congruent to 1 modulo d, we have that the composition of functions k(x) factors. k(x) = (1/d^2)*((1 + x*d^2 + x^2*d^2 - d - 2*x*d + 41*d^2)*(**c**^2*d^2*x^2 + x*d^2*c^2 + 41*c^2*d^2 + 2*x*d*c^2 - 2*x*d*c^2 + c*d - c^2*d + 1). So k(x) is the product of two integers greater than one and is thus composite.

REFERENCES John Stillwell, Elements of Number Theory, Springer 2003, page 3.

LINKS **Matt C. Anderson**, Table of n, a(n) for n = 1..75

EXAMPLE If d = 1 then n = **c***n^2 + (1 - **c**)*x + 41***c** - 1. This is, up to a change of variables, equivalent to A201998.

MAPLE

```
maxn := 1000;
A := {};
for n to maxn do
g := n^2+n+41;
if isprime(g) = false then
A := `union`(A, {n}) :
end if :
end do :
A:
# the A list now contains Positive numbers n such that
# n^2 + n + 41 is composite.
# an upper limit for the number of iterations in the
# triple nested while loops is 1000^3 or a billion.
c:=1:
d:=1:
x:=-1:
p:=41:
q:=c*d*x^2+((d-2)*c+1)*x+((p*d^2-d+1)*c-1)/d;
A2:=A:
while q < maxn do
while q < maxn do
while q < maxn do
  A2:=A2 minus {q}:
  A2:=A2 minus {c*x^(2)+(c+1)*x+c*p}:
  A2:=A2 minus {c*d*x^2-((d-2)*c+1)*x+((p*d^2-d+1)*c-1)/d}:
  x:=x+1:
  q:=c*d*x^2+((d-2)*c+1)*x+((p*d^2-d+1)*c-1)/d:
end do:
c:=c+1:
x:=-1:
q:=c*d*x^2+((d-2)*c+1)*x+((p*d^2-d+1)*c-1)/d:
end do:
d:=d+1:
c:=1:
x:=-1:
q:=c*d*x^2+((d-2)*c+1)*x+((p*d^2-d+1)*c-1)/d:
end do:
A2
```

CROSSREFS Cf. A007634 (numbers n such that n^2 + n + 41 is composite).
Cf. A201998 and A241529 (similar subsequences of A007634).

KEYWORD nonn

AUTHOR _**Matt C. Anderson**_, Jan 08 2014

EXTENSIONS Corrected and edited by _**Matt C. Anderson**_, Jan 23 2014

STATUS approved

A241529 Positive numbers k such that k^2 + k + 41 is composite and there are no positive integers a,**c**,d such that k = **c***a*z^2 + ((((d+2)*(1/3))***c**-2)*a/d+1)*z + ((((367*d^2+d+1)*(1/9))***c**^2-((d+2)*(1/3))***c**+1)*a/d^2 - (((d-1)*(1/3))***c**+1)/d)/**c** for an integer z.

2887, 2969, 3056, 3220, 3365, 3464, 3565, 3611, 3719, 3746, 3814, 3836, 3874, 3879, 3955, 4142, 4147, 4211, 4277, 4371, 4403, 4483, 4564, 4572, 4661, 4730, 4813, 4881, 4888, 4902, 4906, 4965, 4982, 5132, 5175, 5208, 5410, 5431, 5509, 5527, 5564, 5624, 5669 (list; graph; refs; listen; history; edit; text; internal format)

OFFSET 1,1

COMMENTS This sequence has a restriction involving 4 variables. More composite cases are described with a better restrictive expression. The expression for k(a,**c**,d,z) will force k^2 + k + 41 to be either a fraction or a composite number.
The condition on k(a,**c**,d,z) was determined by quadratic curve fitting. It has been automated with the Maple Interactive() command. The ultimate motivation is to try to find a closed-from expression that generates all the composite cases of k^2 + k + 41 for integer k.
What is the smallest value of n where this sequence's a(n) < 2n? (For A194634, this value is 2358.) - J. Lowell, Feb 25 2019

REFERENCES John Stillwell, Elements of Number Theory, Springer, 2003, page 3.

LINKS Table of n, a(n) for n=1..43.
Matt C. Anderson, Graph of composite values for n^2 + n + 41 with a modular symmetry.
Eric Weisstein's World of Mathematics, Prime-Generating Polynomial

MAPLE

```
# Euler considered the prime values for n^2 + n + 41;
# This is a 76 second calculation on a 2.93 GHz machine
h := n^2+n+41;
y := c*a*z^2+((((d+2)*(1/3))*c-2)*a/d+1)*z+((((367*d^2+d+1)*(1/9))*c^2-((d+2)*(1/3))*c+1)*a/d^2-(((d-1)*(1/3))*c+1)/d)/c;
y2 := subs(n = y, h);
y3 := factor(y2);
# note that y is an expression in 4 variables.
# After a composition of functions, an algebraic factorization
# can be observed in y3.  As long as y3 is an integer, it will
# be composite.  This is because y3 factors and both factors
# are integers bigger than one.
maxn := 6000;
A := {}:
for n to maxn do
g := n^2+n+41:
if isprime(g) = false then A := `union`(A, {n}) end if :
end do:
# now the A set contains composite values of the form
# n^2 + n + 41 less than maxn.
c := 1: a := 1: d := 1: z := -1: p := 41:
q := c*a*z^2+((((d+2)*(1/3))*c-2)*a/d+1)*z+((((367*d^2+d+1)*(1/9))*c^2-((d+2)*(1/3))*c+1)*a/d^2-(((d-1)*(1/3))*c+1)/d)/c:
A2 := A:
```

```
while q < maxn do
while `and`(q < maxn, d < 100) do
while q < maxn do while
q < maxn do
A2 := `minus`(A2, {q});
A2 := `minus`(A2, {c*a*z^2+((((d+2)*(1/3))*c-2)*a/d+1)*z+((((367*d^2+d+1)*(1/9))*c^2-((d+2)*(1/3))*c+1)*a/d^2-(((d-1)*(1/3))*c+1)/d)/c});
z := z+1;
A2 := `minus`(A2, {c*a*z^2-((((d+2)*(1/3))*c-2)*a/d+1)*(1*z)+((((367*d^2+d+1)*(1/9))*c^2-((d+2)*(1/3))*c+1)*a/d^2-(((d-1)*(1/3))*c+1)/d)/c}); q
    := c*a*z^2+((((d+2)*(1/3))*c-2)*a/d+1)*z+((((367*d^2+d+1)*(1/9))*c^2-((d+2)*(1/3))*c+1)*a/d^2-(((d-1)*(1/3))*c+1)/d)/c
end do;
a := a+1; z := -1;
q := c*a*z^2+((((d+2)*(1/3))*c-2)*a/d+1)*z+((((367*d^2+d+1)*(1/9))*c^2-((d+2)*(1/3))*c+1)*a/d^2-(((d-1)*(1/3))*c+1)/d)/c :
end do;
d := d+1: a := 1:
q := c*a*z^2+((((d+2)*(1/3))*c-2)*a/d+1)*z+((((367*d^2+d+1)*(1/9))*c^2-((d+2)*(1/3))*c+1)*a/d^2-(((d-1)*(1/3))*c+1)/d)/c :
end do:
c := c+1: d := 1:
q := c*a*z^2+((((d+2)*(1/3))*c-2)*a/d+1)*z+((((367*d^2+d+1)*(1/9))*c^2-((d+2)*(1/3))*c+1)*a/d^2-(((d-1)*(1/3))*c+1)/d)/c :
end do:
A2;
# _Matt C. Anderson_, May 13 2014
```

CROSSREFS Cf. A007634, A055390, A201998, and with division, A235381.

KEYWORD nonn

AUTHOR **_Matt C. Anderson_**, Apr 27 2014

STATUS approved

A248015 Positive numbers n such that n^2 + 1 is composite and there are no positive integers **c** and z such that n = **c***z^2 + z + **c**.

8, 18, 28, 30, 34, 44, 46, 48, 50, 58, 60, 64, 68, 70, 76, 78, 86, 88, 96, 98, 100, 104, 108, 114, 118, 128, 136, 144, 148, 158, 164, 166, 168, 178, 186, 188, 190, 194, 196, 198, 200 (list; graph; refs; listen; history; edit; text; internal format)

OFFSET 1,1

COMMENTS Subset of A134407.

If f(x) = x^2 + 1 and g(**c**,y) = **c***y^2 + y + **c** then the algebraic substitution of g for x gives a factorization: f(g(**c**,y)) = (y^2 + 1)*(**c**^2*y^2 + **c**^2 + 2***c***y + 1). Since both factors of f(g(**c**,y)) are integers greater than one, f(g(**c**,y)) is a composite number.

The numbers are necessarily even terms from A134407 since for odd n = 2c + 1 one has the "forbidden" decomposition with z = 1. - M. F. Hasler, Oct 04 2014

LINKS Table of n, a(n) for n=1..41.

Eric Weisstein's World of Mathematics, Landau's Problems

MAPLE

```
maxn:=200:
mb:=proc(n::integer)::integer;
  if isprime(n^2+1)=false then return n else return -1 fi;
end proc:
A134407 := Vector(maxn):
for a from 1 to maxn do A134407[a]:= mb(a): end do:
A134407s:=convert(A134407, 'set') minus {-1}:
A134407l:=convert(A134407s, 'list'):
for c from 1 to 200 do
  for z from 1 to 20 do
    A134407s := A134407s minus {c*z^2 + z + c}:
  end do:
end do:
A134407s;
```

PROG (PARI) is(n)={!bittest(n, 0)&&!isprime(n^2+1)&&!for(z=2, sqrtint(n), (n-z)%(z^2+1)||return)} \\ M. F. Hasler, Oct 04 2014

CROSSREFS Cf. A134407.

KEYWORD nonn

AUTHOR **_Matt C. Anderson_**, Sep 29 2014

STATUS approved

A000045 Fibonacci numbers: F(n) = F(n-1) + F(n-2) with F(0) = 0 and F(1) = 1. (Formerly M0692 N0256)

+30
5136

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040, 1346269, 2178309, 3524578, 5702887, 9227465, 14930352, 24157817, 39088169, 63245986, 102334155 (list; graph; refs; listen; history; edit; text; internal format)

OFFSET 0,4

COMMENTS Also sometimes called Lamé's sequence.

F(n+2) = number of binary sequences of length n that have no consecutive 0's.

F(n+2) = number of subsets of {1,2,...,n} that contain no consecutive integers.

F(n+1) = number of tilings of a 2 X n rectangle by 2 X 1 dominoes.

F(n+1) = number of matchings (i.e., Hosoya index) in a path graph on n vertices: F(5)=5 because the matchings of the path graph on the vertices A, B, **C**, D are the empty set, {AB}, {BC}, {CD} and {AB, CD}. - Emeric Deutsch, Jun 18 2001

F(n) = number of compositions of n+1 with no part equal to 1. [Cayley, Grimaldi]

Positive terms are the solutions to z = 2*x*y^4 + (x^2)*y^3 - 2*(x^3)*y^2 - y^5 - (x^4)*y + 2*y for x,y >= 0 (Ribenboim, page 193). When x=F(n), y=F(n + 1) and z > 0 then z=F(n + 1).

For Fibonacci search see Knuth, Vol. 3; Horowitz and Sahni; etc.

F(n) is the diagonal sum of the entries in Pascal's triangle at 45 degrees slope. - Amarnath Murthy, Dec 29 2001

F(n+1) is the number of perfect matchings in ladder graph L_n = P_2 X P_n. - Sharon Sela (sharonsela(AT)hotmail.com), May 19 2002

F(n+1) = number of (3412,132)-, (3412,213)- and (3412,321)-avoiding involutions in S_n.

This is also the Horadam sequence (0,1,1,1). - Ross La Haye, Aug 18 2003

An INVERT transform of A019590. INVERT([1,1,2,3,5,8,...]) gives A000129. INVERT([1,2,3,5,8,13,21,...]) gives A028859. - Antti Karttunen, Dec 12 2003

Number of meaningful differential operations of the k-th order on the space R^3. - Branko Malesevic, Mar 02 2004

F(n) = number of compositions of n-1 with no part greater than 2. Example: F(4) = 3 because we have 3 = 1+1+1 = 1+2 = 2+1.

F(n) = number of compositions of n into odd parts; e.g., F(6) counts 1+1+1+1+1+1, 1+1+1+3, 1+1+3+1, 1+3+1+1, 1+5, 3+1+1+1, 3+3, 5+1. - Clark Kimberling, Jun 22 2004

F(n) = number of binary words of length n beginning with 0 and having all runlengths odd; e.g., F(6) counts 010101, 010111, 010001, 011101, 011111, 000101, 000111, 000001. - Clark Kimberling, Jun 22 2004

The number of sequences (s(0),s(1),...,s(n)) such that 0<s(i)<5, |s(i)-s(i-1)|=1 and s(0)=1 is F(n+1); e.g., F(5+1) = 8 corresponds to 121212, 121232, 121234, 123212, 123232, 123234, 123432, 123434. - Clark Kimberling, Jun 22 2004 [corrected by Neven Juric, Jan 09 2009]

Likewise F(6+1) = 13 corresponds to these thirteen sequences with seven numbers: 1212121, 1212123, 1212321, 1212323, 1212343, 1232121, 1232123, 1232321, 1232323, 1232343, 1234321, 1234323, 1234343. - Neven Juric, Jan 09 2008

A relationship between F(n) and the Mandelbrot set is discussed in the link "Le nombre d'or dans l'ensemble de Mandelbrot" (in French). - Gerald McGarvey, Sep 19 2004

For n>0, the continued fraction for F(2n-1)*Phi = [F(2n);L(2n-1),L(2n-1),L(2n-1),...] and the continued fraction for F(2n)*Phi=[F(2n+1)-1;1,L(2n)-2,1,L(2n)-2,...]. Also true: F(2n)*Phi=[F(2n+1);-L(2n),L(2n),-L(2n),L(2n),...] where L(i) is the i-th Lucas number (A000204).... - Clark Kimberling, Nov 28 2004 [corrected by Hieronymus Fischer, Oct 20 2010]

For any nonzero number k, the continued fraction [4,4,...,4,k], which is n 4's and a single k, equals (F(3n) + k*F(3n+3))/(F(3n-3) + k*F(3n)). - Greg Dresden, Aug 07 2019

F(n+1) (for n >= 1) = number of permutations p of 1,2,3,...,n such that |k-p(k)| <= 1 for k=1,2,...,n. (For <= 2 and <= 3, see A002524 and A002526.) - Clark Kimberling, Nov 28 2004

The ratios F(n+1)/F(n) for n > 0 are the convergents to the simple continued fraction expansion of the golden section. - Jonathan Sondow, Dec 19 2004

Lengths of successive words (starting with a) under the substitution: {a -> ab, b -> a}. - Jeroen F.J. Laros, Jan 22 2005

The Fibonacci sequence, like any additive sequence, naturally tends to be geometric with common ratio not a rational power of 10; consequently, for a sufficiently large number of terms, Benford's law of first significant digit (i.e., first digit 1 <= d <= 9 occurring with probability log_10(d+1) - log_10(d)) holds. - Lekraj Beedassy, Apr 29 2005 (See Brown-Duncan, 1970. - N. J. A. Sloane, Feb 12 2017)

a(n) = Sum_{k=0..n} abs(A108299(n, k)). - Reinhard Zumkeller, Jun 01 2005

a(n) = A001222(A000304(n)).

F(n+2) = Sum_{k=0..n} binomial(floor((n+k)/2),k), row sums of A046854. - Paul Barry, Mar 11 2003

Number of order ideals of the "zig-zag" poset. See vol. 1, ch. 3, prob. 23 of Stanley. - Mitch Harris, Dec 27 2005

F(n+1)/F(n) is also the Farey fraction sequence (see A097545 for explanation) for the golden ratio, which is the only number whose Farey fractions and continued fractions are the same. - Joshua Zucker, May 08 2006

a(n+2) is the number of paths through 2 plates of glass with n reflections (reflections occurring at plate/plate or plate/air interfaces). Cf. A006356-A006359. - Mitch Harris, Jul 06 2006

F(n+1) equals the number of downsets (i.e., decreasing subsets) of an n-element fence, i.e., an ordered set of height 1 on {1,2,...,n} with 1 > 2 < 3 > 4 < ... n and no other comparabilities. Alternatively, F(n+1) equals the number of subsets A of {1,2,...,n} with the property that, if an odd k is in A, then the adjacent elements of {1,2,...,n} belong to A, i.e., both k - 1 and k + 1 are in A (provided they are in {1,2,...,n}). - Brian Davey, Aug 25 2006

Number of Kekulé structures in polyphenanthrenes. See the paper by Lukovits and Janezic for details. - Parthasarathy Nambi, Aug 22 2006

Inverse: With phi = (sqrt(5) + 1)/2, round(log_phi(sqrt((sqrt(5) a(n) + sqrt(5 a(n)^2 - 4))(sqrt(5) a(n) + sqrt(5 a(n)^2 + 4)))/2)) = n for n >= 3, obtained by rounding the arithmetic mean of the inverses given in A001519 and A001906. - David W. Cantrell (DWCantrell(AT)sigmaxi.net), Feb 19 2007

A result of Jacobi from 1848 states that every symmetric matrix over a p.i.d. is congruent to a triple-diagonal matrix. Consider the maximal number T(n) of summands in the determinant of an n X n triple-diagonal matrix. This is the same as the number of summands in such a determinant in which the main-, sub- and super-diagonal elements are all nonzero. By expanding on the first row we see that the sequence of T(n)'s is the Fibonacci sequence without the initial stammer on the 1's. - Larry Gerstein (gerstein(AT)math.ucsb.edu), Mar 30 2007

Suppose psi=log(phi). We get the representation F(n)=(2/sqrt(5))*sinh(n*psi) if n is even; F(n)=(2/sqrt(5))*cosh(n*psi) if n is odd. There is a similar representation for Lucas numbers (A000032). Many Fibonacci formulas now easily follow from appropriate sinh and cosh formulas. For example: the de Moivre theorem (cosh(x)+sinh(x))^m = cosh(mx)+sinh(mx) produces L(n)^2 + 5F(n)^2 = 2L(2n) and L(n)F(n) = F(2n) (setting x=n*psi and m=2). - Hieronymus Fischer, Apr 18 2007

Inverse: floor(log_phi(sqrt(5)*F(n)) + 1/2) = n, for n > 1. Also for n > 0, floor((1/2)*log_phi(5*F(n)*F(n+1))) = n. Extension valid for integer n, except n=0,-1: floor((1/2)*sign(F(n)*F(n+1))*log_phi|5*F(n)*F(n+1)|) = n (where sign(x) = sign of x). - Hieronymus Fischer, May 02 2007

F(n+2) = The number of Khalimsky-continuous functions with a two-point codomain. - Shiva Samieinia (shiva(AT)math.su.se), Oct 04 2007

From Kauffman and Lopes, Proposition 8.2, p. 21: "The sequence of the determinants of the Fibonacci sequence of rational knots is the Fibonacci sequence (of numbers)." - Jonathan Vos Post, Oct 26 2007

This is a_1(n) in the Doroslovacki reference.

Let phi = (sqrt(5)+1)/2 = 1.6180339...; then phi^n = (1/phi)*a(n) + a(n+1). Example: phi^4 = 6.8541019... = (0.6180339...)*3 + 5. Also phi = 1/1 + 1/2 + 1/(2*5) + 1/(5*13) + 1/(13*34) + 1/(34*89) + ... - Gary W. Adamson, Dec 15 2007

The sequence of first differences, F(n+1)-F(n), is essentially the same sequence: 1, 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, ... - Colm Mulcahy, Mar 03 2008

a(n) = the number of different ways to run up a staircase with n steps, taking steps of odd sizes where the order is relevant and there is no other restriction on the number or the size of each step taken. - Mohammad K. Azarian, May 21 2008

Equals row sums of triangle A144152. - Gary W. Adamson, Sep 12 2008

Except for the initial term, the numerator of the convergents to the recursion x = 1/(x+1). - Cino Hilliard, Sep 15 2008

F(n) is the number of possible binary sequences of length n that obey the sequential construction rule: if last symbol is 0, add the complement (1); else add 0 or 1. Here 0,1 are metasymbols for any 2-valued symbol set. This rule has obvious similarities to JFJ Laros's rule, but is based on addition rather than substitution and creates a tree rather than a single sequence. - Ross Drewe, Oct 05 2008

F(n) = Product_{k=1..(n-1)/2} (1 + 4*cos^2 k*Pi/n), where terms = roots to the Fibonacci product polynomials, A152063. - Gary W. Adamson, Nov 22 2008

Fp == 5^((p-1)/2) mod p, p = prime [Schroeder, p. 90]. - Gary W. Adamson & Alexander R. Povolotsky, Feb 21 2009

(Ln)^2 - 5*(Fn)^2 = 4*(-1)^n. Example: 11^2 - 5*5 = -4. - Gary W. Adamson, Mar 11 2009

Output of Kasteleyn's formula for the number of perfect matchings of an m X n grid specializes to the Fibonacci sequence for m=2. - Sarah-Marie Belcastro (smbelcas(AT)toroidalsnark.net), Jul 04 2009

(F(n),F(n+4)) satisfies the Diophantine equation: X^2 + Y^2 - 7XY = 9*(-1)^n. - Mohamed Bouhamida, Sep 06 2009

(F(n),F(n+2)) satisfies the Diophantine equation: X^2 + Y^2 - 3XY = (-1)^n. - Mohamed Bouhamida, Sep 08 2009

a(n+2) = A083662(A131577(n)). - Reinhard Zumkeller, Sep 26 2009

Difference between number of closed walks of length n+1 from a node on a pentagon and number of walks of length n+1 between two adjacent nodes on a pentagon. - Henry Bottomley, Feb 10 2010

F(n+1) = number of Motzkin paths of length n having exactly one weak ascent. A Motzkin path of length n is a lattice path from (0,0) to (n,0) consisting of U=(1,1), D=(1,-1) and H=(1,0) steps and never going below the x-axis. A weak ascent in a Motzkin path is a maximal sequence of consecutive U and H steps. Example: a(5)=5 because we have (HHHH), (HHU)D, (HUH)D, (UHH)D, and (UU)DD (the unique weak ascent is shown between parentheses; see A114690). - Emeric Deutsch, Mar 11 2010

(F(n-1) + F(n+1))^2 - 5F(n-2)*F(n+2) = 9*(-1)^n. - Mohamed Bouhamida, Mar 31 2010

From the Pinter and Ziegler reference's abstract: authors "show that essentially the Fibonacci sequence is the unique binary recurrence which contains infinitely many three-term arithmetic progressions. A criterion for general linear recurrences having infinitely many three-term arithmetic progressions is also given." - Jonathan Vos Post, May 22 2010

F(n+1) = number of paths of length n starting at initial node on the path graph P_4. - Johannes W. Meijer, May 27 2010

F(k) = Number of cyclotomic polynomials in denominator of generating function for number of ways to place k nonattacking queens on an n X n board. - Vaclav Kotesovec, Jun 07 2010

As n-> inf., (a(n)/a(n-1) - a(n-1)/a(n)) tends to 1.0. Example: a(12)/a(11) - a(11)/a(12) = 144/89 - 89/144 = 0.99992197.... - Gary W. Adamson, Jul 16 2010

From Hieronymus Fischer, Oct 20 2010: (Start)

Fibonacci numbers are those numbers m such that m*phi is closer to an integer than k*phi for all k, 1<=k<m. More formally: a(0)=0, a(1)=1, a(2)=1, a(n+1)=minimal m>a(n) such that m*phi is closer to an integer than a(n)*phi.

For all numbers 1 <= k < F(n), the inequality |k*phi-round(k*phi)| > |F(n)*phi-round(F(n)*phi)| holds.

F(n)*phi - round(F(n)*phi) = -((-phi)^(-n)), for n > 1.

Fract(1/2 + F(n)*phi) = 1/2 -(-phi)^(-n), for n > 1.

Fract(F(n)*phi) = (1/2)*(1 + (-1)^n) - (-phi)^(-n), n > 1.

Inverse: n = -log_phi |1/2 - fract(1/2 + F(n)*phi)|.

(End)

F(A001177(n)*k) mod n = 0, for any integer k. - Gary Detlefs, Nov 27 2010

F(n+k)^2 - F(n)^2 = F(k)*F(2n+k), for even k. - Gary Detlefs, Dec 04 2010

F(n+k)^2 + F(n)^2 = F(k)*F(2n+k), for odd k. - Gary Detlefs, Dec 04 2010

F(n) = round(phi* F(n-1)) for n > 1. - Joseph P. Shoulak, Jan 13 2012

For n > 0: a(n) = length of n-th row in Wythoff array A003603. - Reinhard Zumkeller, Jan 26 2012

From Bridget Tenner, Feb 22 2012: (Start)

The number of free permutations of [n].

The number of permutations of [n] for which s_k in supp(w) implies s_{k+-1} not in supp(w).

The number of permutations of [n] in which every decomposition into length(w) reflections is actually composed of simple reflections. (End)

The sequence F(n+1)^(1/n) is increasing. The sequence F(n+2)^(1/n) is decreasing. - Thomas Ordowski, Apr 19 2012

Two conjectures: For n > 1, F(n+2)^2 mod F(n+1)^2 = F(n)*F(n+1) - (-1)^n. For n > 0, (F(2n) + F(2n+2))^2 = F(4n+3) + Sum_{k = 2..2n} F(2k). - Alex Ratushnyak, May 06 2012

From Ravi Kumar Davala, Jan 30 2014: (Start)

Proof of Ratushnyak's first conjecture: For n > 1, F(n+2)^2 - F(n)*F(n+1) + (-1)^n = 2F(n+1)^2.

Consider: F(n+2)^2 - F(n)*F(n+1) - 2F(n+1)^2
        = F(n+2)^2 - F(n+1)^2 - F(n+1)^2 - F(n)*F(n+1)
        = (F(n+2) + F(n+1))*(F(n+2) - F(n+1)) - F(n+1)*(F(n+1) + F(n))
        = F(n+3)*F(n) - F(n+1)*F(n+2) = -(-1)^n.

Proof of second conjecture: L(n) stands for Lucas number sequence from A000032.

Consider the fact that
    L(2n+1)^2 = L(4n+2) - 2
   (F(2n) + F(2n+2))^2 = F(4n+1) + F(4n+3) - 2

(F(2n) + F(2n+2))^2 = (Sum_{k = 2..2n} F(2k)) + F(4n+3).

(End)

The relationship: INVERT transform of (1,1,0,0,0,...) = (1, 2, 3, 5, 8, ...), while the INVERT transform of (1,0,1,0,1,0,1,...) = (1, 1, 2, 3, 5, 8, ...) is equivalent to: The numbers of compositions using parts 1 and 2 is equivalent to the numbers of compositions using parts == 1 mod 2 (i.e., the odd integers). Generally, the numbers of compositions using parts 1 and k is equivalent to the numbers of compositions of (n+1) using parts 1 mod k. Cf. A000930 for k = 3 and A003269 for k = 4. Example: for k = 2, n = 4 we have the compositions (22; 211, 121; 112; 1111) = 5; but using parts 1 and 3 we have for n = 5: (311, 131, 113, 11111, 5) = 5. - Gary W. Adamson, Jul 05 2012

The sequence F(n) is the binomial transformation of the alternating sequence (-1)^(n-1)*F(n), whereas the sequence F(n+1) is the binomial transformation of the alternating sequence (-1)^n*F(n-1). Both of these facts follow easily from the equalities a(n;1)=F(n+1) and b(n;1)=F(n) where a(n;d) and b(n;d) are so-called "delta-Fibonacci" numbers as defined in comments to A014445 (see also the papers of Witula et al.). - Roman Witula, Jul 24 2012

F(n) is the number of different (n-1)-digit binary numbers such that all substrings of length > 1 have at least one digit equal to 1. Example: for n = 5 there are 8 binary numbers with n - 1 = 4 digits (1000, 1001, 1010, 1011, 1100, 1101, 1110, 1111), only the F(n) = 5 numbers 1010, 1011, 1101, 1110 and 1111 have the desired property. - Hieronymus Fischer, Nov 30 2012

For positive n, F(n+1) equals the determinant of the n X n tridiagonal matrix with 1's along the main diagonal, i's along the superdiagonal and along the subdiagonal where i = sqrt(-1). Example: Det([1,i,0,0; i,1,i,0; 0,i,1,i; 0,0,i,1]) = F(4+1) = 5. - Philippe Deléham, Feb 24 2013

For n >= 1, number of compositions of n where there is a drop between every second pair of parts, starting with the first and second part; see example. Also, a(n+1) is the number of compositions where there is a drop between every second pair of parts, starting with the second and third part; see example. - Joerg Arndt, May 21 2013

Central terms of triangles in A162741 and A208245, n > 0. - Reinhard Zumkeller, Jul 28 2013

For n >= 4, F(n-1) is the number of simple permutations in the geometric grid class given in A226433. - Jay Pantone, Sep 08 2013

a(n) are the pentagon (not pentagonal) numbers because the algebraic degree 2 number rho(5) = 2*cos(Pi/5) = phi (golden section), the length ratio diagonal/side in a pentagon, has minimal polynomial **C**(5,x) = x^2 - x - 1 (see A187360, n=5), hence rho(5)^n = a(n-1)*1 + a(n)*rho(5), n >= 0, in the power basis of the algebraic number field Q(rho(5)). One needs a(-1) = 1 here. See also the P. Steinbach reference under A049310. - Wolfdieter Lang, Oct 01 2013

A010056(a(n)) = 1. - Reinhard Zumkeller, Oct 10 2013

Define F(-n) to be F(n) for n odd and -F(n) for n even. Then for all n and k, F(n+2k)^2 - F(n)^2 = F(n+k)*(F(n+3k) - F(n-k)). - Charlie Marion, Dec 20 2013

(F(n), F(n+2k)) satisfies the Diophantine equation: X^2 + Y^2 - L(2k)*X*Y = F(4k)^2*(-1)^n. This generalizes Bouhamida's comments dated Sep 06 2009 and Sep 08 2009. - Charlie Marion, Jan 07 2014

For any prime p there is an infinite periodic subsequence within F(n) divisible by p, that begins at index n = 0 with value 0, and its first nonzero term at n = A001602(i), and period k = A001602(i). Also see A236479. - Richard R. Forberg, Jan 26 2014

Range of row n of the circular Pascal array of order 5. - Shaun V. Ault, May 30 2014 [orig. Kicey-Klimko 2011, and observations by Glen Whitehead; more general work found in Ault-Kicey 2014]

Nonnegative range of the quintic polynomial 2*y - y^5 + 2*x*y^4 + x^2*y^3 - 2*x^3*y^2 - x^4*y with x, y >= 0, see Jones 1975. - Charles R Greathouse IV, Jun 01 2014

The expression round(1/(F(k+1)/F(n) + F(k)/F(n+1))), for n > 0, yields a Fibonacci sequence with k-1 leading zeros (with rounding 0.5 to 0). - Richard R. Forberg, Aug 04 2014

Conjecture: For n > 0, F(n) is the number of all admissible residue classes for which specific finite subsequences of the Collatz 3n + 1 function consists of n+2 terms. This has been verified for 0 < n < 51. For details see Links. - Mike Winkler, Oct 03 2014

a(4)=3 and a(6)=8 are the only Fibonacci numbers that are of the form prime+1. - Emmanuel Vantieghem, Oct 02 2014

a(1)=1=a(2), a(3)=2 are the only Fibonacci numbers that are of the form prime-1. - Emmanuel Vantieghem, Jun 07 2015

Any consecutive pair (m, k) of the Fibonacci sequence a(n) illustrates a fair equivalence between m miles and k kilometers. For instance, 8 miles ~ 13 km; 13 miles ~ 21 km. -Lekraj Beedassy, Oct 06 2014

Lim_{n -> oo} (log F(n+1)/log F(n))^n = e. - Thomas Ordowski, Oct 06 2014

a(n+1) counts closed walks on K_2, containing one loop on the other vertex. Equivalently the (1,1)_entry of A^(n+1) where the adjacency matrix of digraph is A=(0,1; 1,1). - David Neil McGrath, Oct 29 2014

a(n-1) counts closed walks on the graph G(1-vertex;1-loop,2-loop). - David Neil McGrath, Nov 26 2014

From Tom Copeland, Nov 02 2014: (Start)

Let P(x) = x/(1+x) with comp. inverse Pinv(x) = x/(1-x) = -P[-x], and **C**(x)= [1-sqrt(1-4x)]/2, an o.g.f. for the shifted Catalan numbers A000108, with inverse Cinv(x) = x * (1-x).

Fin(x) = P[**C**(x)] = **C**(x)/[1 + **C**(x)] is an o.g.f. for the Fine numbers, A000957 with inverse Fin^(-1)(x) = Cinv[Pinv(x)] = Cinv[-P(-x)].

Mot(x) = **C**[P(x)] = **C**[-Pinv(-x)] gives an o.g.f. for shifted A005043, the Motzkin or Riordan numbers with comp. inverse Mot^(-1)(x) = Pinv[Cinv(x)] = (x - x^2) / (1 - x + x^2) (cf. A057078).

BTC(x) = **C**[Pinv(x)] gives A007317, a binomial transform of the Catalan numbers, with BTC^(-1)(x) = P[Cinv(x)].

Fib(x) = -Fin[Cinv(Cinv(-x))] = -P[Cinv(-x)] = x + 2 x^2 + 3 x^3 + 5 x^4 + ... = (x+x^2)/[1-x-x^2] is an o.g.f. for the shifted Fibonacci sequence A000045, so the comp. inverse is Fib^(-1)(x) = -**C**[Pinv(-x)] = -BTC(-x) and Fib(x) = -BTC^(-1)(-x).

Generalizing to P(x,t) = x /(1 + t*x) and Pinv(x,t) = x /(1 - t*x) = -P(-x,t) gives other relations to lattice paths, such as the o.g.f. for A091867, **C**[P[x,1-t]], and that for A104597, Pinv[Cinv(x),t+1].

(End)

In keeping with historical accounts (see the references by P. Singh and S. Kak), the generalized Fibonacci sequence a, b, a + b, a + 2b, 2a + 3b, 3a + 5b, ... can also be described as the Gopala-Hemachandra numbers H(n) = H(n-1) + H(n-2), with F(n) = H(n) for a = b = 1, and Lucas sequence L(n) = H(n) for a = 2, b = 1. - Lekraj Beedassy, Jan 11 2015

D. E. Knuth writes: "Before Fibonacci wrote his work, the sequence F_{n} had already been discussed by Indian scholars, who had long been interested in rhythmic patterns that are formed from one-beat and two-beat notes. The number of such rhythms having n beats altogether is F_{n+1}; therefore both Gopāla (before 1135) and Hemachandra (**c**. 1150) mentioned the numbers 1, 2, 3, 5, 8, 13, 21, ... explicitly." (TAOCP Vol. 1, 2nd ed.) - Peter Luschny, Jan 11 2015

F(n+1) equals the number of binary words of length n avoiding runs of zeros of odd lengths. - Milan Janjic, Jan 28 2015

From Russell Jay Hendel, Apr 12 2015: (Start)

We prove Conjecture 1 of Rashid listed in the Formula section.

We use the following notation: F(n)=A000045(n), the Fibonacci numbers, and L(n) = A000032(n), the Lucas numbers. The fundamental Fibonacci-Lucas recursion asserts that G(n) = G(n-1)+ G(n-2), with "L" or "F" replacing "G".

We need the following prerequisites which we label (A), (B),(**C**), (D). The prerequisites are formulas in the Koshy book listed in the References section. (A) F(m-1)+F(m+1) = L(m) (Koshy, p. 97, #32), (B) L(2m)+2(-1)^m = L(m)^2 (Koshy p. 97, #41), (**C**) F(m+k)F(m-k) = (-1)^n F(k)^2 (Koshy, p. 113, #24, Tagiuri's identity), and (D) F(n)^2+F(n+1)^2 = F(2n+1) (Koshy, p. 97, #30).

We must also prove (E), L(n+2) F(n-1) = F(2n+1)+2(-1)^n. To prove (E), first note that by (A), proof of (E) is equivalent to proving that F(n+1)F(n-1) + F(n+3)F(n-1) = F(2n+1)+2(-1)^n. But by (**C**) with k=1, we have F(n+1)F(n-1) = F(n)^2 +(-1)^n. Applying (**C**) again with k=2 and m=n+1, we have F(n+3)F(n-1) = F(n+1)+(-1)^n. Adding these two applications of (**C**) together and using (D) we have, F(n+1)F(n-1) + F(n+3)F(n-1) = F(n)^2 + F(n+1)^2 + 2(-1)^n = F(2n+1)+2(-1)^n, completing the proof of (E).

We now prove Conjecture 1. By (A) and the Fibonacci-Lucas recursion, we have F(2n+1)+F(2n+2)+F(2n+3)+F(2n+4) = [F(2n+1)+F(2n+3)] + [F(2n+2)+F(2n+4)] = L(2n+2)+L(2n+3)=L(2n+4). But then by (B), with m=2n+4, we have sqrt(L(2n+4)+2(-1)^n)) = L(n+2). Finally by (E), we have L(n+2) F(n-1)= F(2n+1)+2*(-1)^n. Dividing both sides by F(n-1), we have (F(2n+1)+2*(-1)^n)/F(n-1) = L(n+2) = sqrt(F(2n+1)+F(2n+2)+F(2n+3)+F(2n+4)+2(-1)^n), as required.

(End)

In Fibonacci's Liber Abaci the rabbit problem appears in the translation of L. E. Sigler on pp. 404-405, and a remark [27] on p. 637. - Wolfdieter Lang, Apr 17 2015

a(n) counts partially ordered partitions of (n-1) into parts 1,2,3 where only the order of adjacent 1's and 2's are unimportant. (See example.) - David Neil McGrath, Jul 27 2015

F(n) divides F(nk). Proved by Marjorie Bicknell and Verner E Hoggatt Jr. - Juhani Heino, Aug 24 2015

F(n) is the number of UDU-equivalence classes of ballot paths of length n. Two ballot paths of length n with steps U = (1,1), D = (1,-1) are UDU-equivalent whenever the positions of UDU are the same in both paths. - Kostas Manes, Aug 25 2015

Cassini's identity F(2n+1) * F(2n+3) = F(2n+2)^2 + 1 is the basis for a geometrical paradox (or dissection fallacy) in A262342. - Jonathan Sondow, Oct 23 2015

For n >= 4, F(n) is the number of up-down words on alphabet {1,2,3} of length n-2. - Ran Pan, Nov 23 2015

F(n+2) is the number of terms in p(n), where p(n)/q(n) is the n-th convergent of the formal infinite continued fraction [a(0),a(1),...]; e.g., p(3) = a(0)a(1)a(2)a(3) + a(0)a(1) + a(0)a(3) + a(2)a(3) + 1 has F(5) terms. Also, F(n+1) is the number of terms in q(n). - Clark Kimberling, Dec 23 2015

F(n+1) (for n >= 1) is the permanent of an n X n matrix M with M(i,j)=1 if |i-j| <= 1 and 0 otherwise. - Dmitry Efimov, Jan 08 2016

A trapezoid has three sides of lengths in order F(n), F(n+2), F(n). For increasing n a very close approximation to the maximum area will have the fourth side equal to 2*F(n+1). For a trapezoid with lengths of sides in order F(n+2), F(n), F(n+2), the fourth side will be F(n+3). - J. M. Bergot, Mar 17 2016

(1) Join two triangles with lengths of sides L(n), F(n+3), L(n+2) and F(n+2), L(n+1), L(n+2) (where L(n)=A000032(n)) along the common side of length L(n+2) to create an irregular quadrilateral. Its area is approximately (5*F(2*n-1) - (F(2*n-7) - F(2*n-13))/5. (2) Join two triangles with lengths of sides L(n), F(n+2), F(n+3) and L(n+1), F(n+1, F(n+3) along the common side F(n+3) to form an irregular quadrilateral. Its area is approximately 4*F(2*n-1) - 2*(F(2*n-7) + F(2*n-18)). - J. M. Bergot, Apr 06 2016

From Clark Kimberling, Jun 13 2016: (Start)

Let T* be the infinite tree with root 0 generated by these rules: if p is in T*, then p+1 is in T* and x*p is in T*.

Let g(n) be the set of nodes in the n-th generation, so that g(0) = {0}, g(1) = {1}, g(2) = {2, x}, g(3) = {3, 2x, x+1, x^2}, etc.

Let T(r) be the tree obtained by substituting r for x.

If a positive integer N is not a square and r = sqrt(N), then the number of (not necessarily distinct) integers in g(n) is A000045(n), for n > = 1. See A274142. (End)

Consider the partitions of n, with all summands initially listed in nonincreasing order. Freeze all the 1's in place and then allow all the other summands to change their order, without displacing any of the 1's. The resulting number of arrangements is a(n+1). - Gregory L. Simay, Jun 14

2016

Limit of the matrix power M^k shown in A163733, Sep 14 2016; as k-->inf. results in a single column vector equal to the Fibonacci sequence. - Gary W. Adamson, Sep 19 2016

F(n) and Lucas numbers L(n), being related by the formulas F(n) = (F(n-1) + L(n-1))/2 and L(n) = 2 F(n+1) - F(n), are a typical pair of "autosequences" (see the link to OEIS Wiki). - Jean-François Alcover, Jun 10 2017

Also the number of independent vertex sets and vertex covers in the (n-2)-path graph. - Eric W. Weisstein, Sep 22 2017

Shifted numbers of {UD, DU, FD, DF}-equivalence classes of Łukasiewicz paths. Łukasiewicz paths are P-equivalent iff the positions of pattern P are identical in these paths. - Sergey Kirgizov, Apr 08 2018

For n>0, F(n) = the number of Markov equivalence classes with skeleton the path on n nodes. See Theorem 2.1 in the article by A. Radhakrishnan et al. below. - Liam Solus, Aug 23 2018

For n >= 2, also: number of terms in A032858 (every other base-3 digit is strictly smaller than its neighbors) with n-2 digits in base 3. - M. F. Hasler, Oct 05 2018

F(n+1) is the number of fixed points of the Foata transformation on S_n. - Kevin Long, Oct 17 2018

F(n+2) is the dimension of the Hecke algebra of type A_n with independent parameters (0,1,0,1,...) or (1,0,1,0,...). See Corollary 1.5 in the link "Hecke algebras with independent parameters". - Jia Huang, Jan 20 2019

The sequence is the second INVERT transform of (1, -1, 2, -3, 5, -8, 13, ...) and is the first sequence in an infinite set of successive INVERT transforms generated from (1, 0, 1, 0, 1, ...). Refer to the array shown in A073133. - Gary W. Adamson, Jul 16 2019

From Kai Wang, Dec 16 2019: (Start)
F(n*k)/F(k) = Sum_{i=0..n-1; j=0..n-1; i+2*j=n-1} (-1)^(j*(k-1))*L(k)^i*((i+j)!/(i!*j!)).
F((2*m+1)*k)/F(k) = Sum_{i=0..m-1} (-1)^(i*k)*L((2*m-2*i)*k) + (-1)^(m*k).
F(2*m*k)/F(k) = Sum_{i=0..m-1} (-1)^{i*k}*L((2*m-2*i-1)*k}.
F(m+s)*F(n+r) - F(m+r)*F(n+s) = (-1)^{n+s}*F(m-n)*F(r-s).
F(m+r)*F(n+s) + F(m+s)*F(n+r) = (2*L(m+n+r+s) - (-1)^{n+s}L(m-n)L(r-s))/5.
L(m+r)*L(n+s) - 5*F(m+s)*F(n+r) = (-1)^{n+s}*L(m-n)*L(r-s).
L(m+r)*L(n+s) + 5*F(m+s)*F(n+r) = 2*L(m+n+r+s) + (-1)^{n+s}*5*F(m-n)*F(r-s) .
L(m+r)*L(n+s) - L(m+s)*L(n+r) = (-1)^{n+s}*5*F(m-n)*F(r-s). (End)

F(n+1) is the number of permutations in S_n whose principal order ideals in the weak order are Boolean lattices. - Bridget Tenner, Jan 16 2020

F(n+1) is the number of permutations w in S_n that form Boolean intervals [s, w] in the weak order for every simple reflection s in the support of w. - Bridget Tenner, Jan 16 2020

F(n+1) is the number of subsets of {1,2,..,n} in which all differences between successive elements of subsets are odd. For example, for n = 6, F(7) = 13 and the 13 subsets are {6}, {1,6}, {3,6}, {5,6}, {2,3,6}, {2,5,6}, {4,5,6}, {1,2,3,6}, {1,2,5,6}, {1,4,5,6}, {3,4,5,6}, {2,3,4,5,6}, {1,2,3,4,5,6}. For even differences between elements see Comment in A016116. - Enrique Navarrete, Jul 01 2020

F(n) is the number of subsets of {1,2,..,n} in which the smallest element of the subset equals the size of the subset (this type of subset is sometimes called extraordinary). For example, F(6) = 8 and the subsets are {1}, {2,3}, {2,4}, {2,5}, {3,4,5}, {2,6}, {3,4,6}, {3,5,6}. It is easy to see that these subsets follow the Fibonacci recursion F(n) = F(n-1) + F(n-2) since we get F(n) such subsets by keeping all F(n-1) subsets from the previous stage (in the example, the F(5)=5 subsets that don't include 6), and by adding one to all elements and appending an additional element n to each subset in F(n-2) subsets (in the example, by applying this to the F(4)=3 subsets {1}, {2,3}, {2,4} we obtain {2,6}, {3,4,6}, {3,5,6}). - Enrique Navarrete, Sep 28 2020

Named "série de Fibonacci" by Lucas (1877) after the Italian mathematician Fibonacci (Leonardo Bonacci, **c.** 1170 - **c.** 1240/50). In 1876 he named the sequence "série de Lamé" after the French mathematician Gabriel Lamé (1795 - 1870). - Amiram Eldar, Apr 16 2021

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Index entries for "core" sequences
Index to divisibility sequences
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Index entries for linear recurrences with constant coefficients, signature (1,1).
Index entries for two-way infinite sequences
Index entries for sequences related to Benford's law

FORMULA

G.f.: x / (1 - x - x^2).
G.f.: Sum_{n>=0} x^n * Product_{k=1..n} (k + x)/(1 + k*x). - Paul D. Hanna, Oct 26 2013
F(n) = ((1+sqrt(5))^n - (1-sqrt(5))^n)/(2^n*sqrt(5)).
Alternatively, F(n) = ((1/2+sqrt(5)/2)^n - (1/2-sqrt(5)/2)^n)/sqrt(5).
F(n) = F(n-1) + F(n-2) = -(-1)^n F(-n).
F(n) = round(phi^n/sqrt(5)).
F(n+1) = Sum_{j=0..floor(n/2)} binomial(n-j, j).
A strong divisibility sequence, that is, gcd(a(n), a(m)) = a(gcd(n, m)) for all positive integers n and m. - Michael Somos, Jan 03 2017
E.g.f.: (2/sqrt(5))*exp(x/2)*sinh(sqrt(5)*x/2). - Len Smiley, Nov 30 2001
[0 1; 1 1]^n [0 1] = [F(n); F(n+1)]
x | F(n) ==> x | F(kn).
A sufficient condition for F(m) to be divisible by a prime p is (p - 1) divides m, if p == 1 or 4 (mod 5); (p + 1) divides m, if p == 2 or 3 (mod 5); or 5 divides m, if p = 5. (This is essentially Theorem 180 in Hardy and Wright.) - Fred W. Helenius (fredh(AT)ix.netcom.com), Jun 29 2001
a(n)=F(n) has the property: F(n)*F(m) + F(n+1)*F(m+1) = F(n+m+1). - Miklos Kristof, Nov 13 2003
From Kurmang. Aziz. Rashid, Feb 21 2004: (Start)
Conjecture 1: for n >= 2, sqrt(F(2n+1) + F(2n+2) + F(2n+3) + F(2n+4) + 2*(-1)^n) = (F(2n+1) + 2*(-1)^n)/F(n-1). [For a proof see Comments section.]
Conjecture 2: for n >= 0, (F(n+2)*F(n+3)) - (F(n+1)*F(n+4)) + (-1)^n = 0.
[Two more conjectures removed by Peter Luschny, Nov 17 2017]
Theorem 1: for n >= 0, (F(n+3)^ 2 - F(n+1)^ 2)/F(n+2) = (F(n+3)+ F(n+1)).
Theorem 2: for n >= 0, F(n+10) = 11*F(n+5) + F(n).
Theorem 3: for n >= 6, F(n) = 4*F(n-3) + F(n-6). (End)
Conjecture 2 of Rashid is actually a special case of the general law F(n)*F(m) + F(n+1)*F(m+1) = F(n+m+1) (take n <- n+1 and m <- -(n+4) in this law). - Harmel Nestra (harmel.nestra(AT)ut.ee), Apr 22 2005
Conjecture 2 of Rashid Kurmang simplified: F(n)*F(n+3) = F(n+1)*F(n+2)-(-1)^n. Follows from d'Ocagne's identity: m=n+2. - Alex Ratushnyak, May 06 2012
Conjecture: for all **c** such that 2-Phi <= **c** < 2*(2-Phi) we have F(n) = floor(Phi*a(n-1)+**c**) for n > 2. - Gerald McGarvey, Jul 21 2004
|2*F(n) - 9*F(n+1)| = 4*A000032(n) + A000032(n+1). - Creighton Dement, Aug 13 2004
For x > Phi, Sum_{n>=0} F(n)/x^n = x/(x^2 - x - 1) - Gerald McGarvey, Oct 27 2004
F(n+1) = exponent of the n-th term in the series f(x, 1) determined by the equation f(x, y) = xy + f(xy, x). - Jonathan Sondow, Dec 19 2004
a(n-1) = Sum_{k=0..n} (-1)^k*binomial(n-ceiling(k/2), floor(k/2)). - Benoit Cloitre, May 05 2005
F(n+1) = Sum_{k=0..n} binomial((n+k)/2, (n-k)/2)(1+(-1)^(n-k))/2. - Paul Barry, Aug 28 2005
Fibonacci(n) = Product_{j=1..ceiling(n/2)-1} (1 + 4(cos(j*Pi/n))^2). [Bicknell and Hoggatt, pp. 47-48.] - Emeric Deutsch, Oct 15 2006
F(n) = 2^-(n-1)*Sum_{k=0..floor((n-1)/2)} binomial(n,2*k+1)*5^k. - Hieronymus Fischer, Feb 07 2006
a(n) = (b(n+1) + b(n-1))/n where {b(n)} is the sequence A001629. - Sergio Falcon, Nov 22 2006
F(n*m) = Sum_{k = 0..m} binomial(m,k)*F(n-1)^k*F(n)^(m-k)*F(m-k). The generating function of F(n*m) (n fixed, m = 0,1,2,...) is G(x) = F(n)*x / ((1 - F(n-1)*x)^2 - F(n)*x*(1 - F(n-1)*x) - (F(n)*x)^2). E.g., F(15) = 610 = F(5*3) = binomial(3,0)* F(4)^0*F(5)^3*F(3) + binomial(3,1)* F(4)^1*F(5)^2*F(2) + binomial(3,2)* F(4)^2*F(5)^1*F(1) + binomial(3,3)* F(4)^3*F(5)^0*F(0) = 1*1*125*2 + 3*3*25*1 + 3*9*5*1 + 1*27*1*0 = 250 + 225 + 135 + 0 = 610. - Miklos Kristof, Feb 12 2007
From Miklos Kristof, Mar 19 2007: (Start)
Let L(n) = A000032(n) = Lucas numbers. Then:
For a >= b and odd b, F(a+b) + F(a-b) = L(a)*F(b).
For a >= b and even b, F(a+b) + F(a-b) = F(a)*L(b).
For a >= b and odd b, F(a+b) - F(a-b) = F(a)*L(b).
For a >= b and even b, F(a+b) - F(a-b) = L(a)*F(b).
F(n+m) + (-1)^m*F(n-m) = F(n)*L(m);
F(n+m) - (-1)^m*F(n-m) = L(n)*F(m);
F(n+m+k) + (-1)^k*F(n+m-k) + (-1)^m*(F(n-m+k) + (-1)^k*F(n-m-k)) = F(n)*L(m)*L(k);
F(n+m+k) - (-1)^k*F(n+m-k) + (-1)^m*(F(n-m+k) - (-1)^k*F(n-m-k)) = L(n)*L(m)*F(k);
F(n+m+k) + (-1)^k*F(n+m-k) - (-1)^m*(F(n-m+k) + (-1)^k*F(n-m-k)) = L(n)*F(m)*L(k);
F(n+m+k) - (-1)^k*F(n+m-k) - (-1)^m*(F(n-m+k) - (-1)^k*F(n-m-k)) = 5*F(n)*F(m)*F(k). (End)
A corollary to Kristof 2007 is 2*F(a+b) = F(a)*L(b) + L(a)*F(b). - Graeme McRae, Apr 24 2014
For n > m, the sum of the 2m consecutive Fibonacci numbers F(n-m-1) thru F(n+m-2) is F(n)*L(m) if m is odd, and L(n)*F(m) if m is even (see the McRae link). - Graeme McRae, Apr 24 2014.
F(n) = b(n) + (p-1)*Sum_{k=2..n-1} floor(b(k)/p)*F(n-k+1) where b(k) is the digital sum analog of the Fibonacci recurrence, defined by b(k) = ds_p(b(k-1)) + ds_p(b(k-2)), b(0)=0, b(1)=1, ds_p=digital sum base p. Example for base p=10: F(n) = A010077(n) + 9*Sum_{k=2..n-1} A059995(A010077(k))*F(n-k+1). - Hieronymus Fischer, Jul 01 2007
F(n) = b(n)+p*Sum_{k=2..n-1} floor(b(k)/p)*F(n-k+1) where b(k) is the digital product analog of the Fonacci recurrence, defined by b(k) = dp_p(b(k-1)) + dp_p(b(k-2)), b(0)=0, b(1)=1, dp_p=digital product base p. Example for base p=10: F(n) = A074867(n) + 10*Sum_{k=2..n-1} A059995(A074867(k))*F(n-k+1). - Hieronymus Fischer, Jul 01 2007
a(n) = denominator of continued fraction [1,1,1,...] (with n ones); e.g., 2/3 = continued fraction [1,1,1]; where barover[1] = [1,1,1,...] = 0.6180339.... - Gary W. Adamson, Nov 29 2007

F(n + 3) = 2F(n + 2) - F(n), F(n + 4) = 3F(n + 2) - F(n), F(n + 8) = 7F(n + 4) - F(n), F(n + 12) = 18F(n + 6) - F(n). - Paul Curtz, Feb 01 2008

1 = 1/(1*2) + 1/(1*3) + 1/(2*5) + 1/(3*8) + 1/(5*13) + ... = 1/2 + 1/3 + 1/10 + 1/24 + 1/65 + 1/168 + ...; where A059929 = (0, 2, 3, 10, 24, 65, 168, ...). - Gary W. Adamson, Mar 16 2008

a(2^n) = Product_{i=0..n-2} B(i) where B(i) is A001566. Example 3*7*47 = F(16). - Kenneth J Ramsey, Apr 23 2008

F(n) = (1/(n-1)!) * (n^(n-1) - (C(n-2,0) + 4*C(n-2,1) + 3*C(n-2,2))*n^(n-2) + (10*C(n-3,0) + 49*C(n-3,1) + 95*C(n-3,2) + 83*C(n-3,3) + 27*C(n-3,4))*n^(n-3) - (90*C(n-4,0) + 740*C(n-4,1) + 2415*C(n-4,2) + 4110*C(n-4,3) + 3890*C(n-4,4) + 1950*C(n-4,5) + 405*C(n-4,6))*n^(n-4) + ...). - André F. Labossière, Nov 24 2004

a(n+1) = Sum_{k=0..n} A109466(n,k)*(-1)^(n-k). -Philippe Deléham, Oct 26 2008

a(n) = Sum_{l_1=0..n+1} Sum_{l_2=0..n}...Sum_{l_i=0..n-i}... Sum_{l_n=0..1} delta(l_1,l_2,...,l_i,...,l_n), where delta(l_1,l_2,...,l_i,...,l_n) = 0 if any l_i + l_(i+1) >= 2 for i=1..n-1 and delta(l_1,l_2,...,l_i,...,l_n) = 1 otherwise. - Thomas Wieder, Feb 25 2009

a(n+1) = 2^n sqrt(Product_{k=1..n} cos(k Pi/(n+1))^2+1/4)) (Kasteleyn's formula specialized). - Sarah-Marie Belcastro (smbelcas(AT)toroidalsnark.net), Jul 04 2009

a(n+1) = Sum_{k=floor(n/2) mod 5} C(n,k) - Sum_{k=floor((n+5)/2) mod 5} C(n,k) = A173125(n) - A173126(n) = |A054877(n)-A052964(n-1)|. - Henry Bottomley, Feb 10 2010

If p[i] = modp(i,2) and if A is Hessenberg matrix of order n defined by: A[i,j] = p[j-i+1], (i <= j), A[i,j]=-1, (i=j+1), and A[i,j]=0 otherwise. Then, for n >= 1, a(n)=det A. - Milan Janjic, May 02 2010

Lim_{k->infinity} F(k+n)/F(k) = (L(n) + F(n)*sqrt(5))/2 with the Lucas numbers L(n)= A000032(n). - Johannes W. Meijer, May 27 2010

For n >= 1, F(n) = round(log_2(2^(phi*F(n-1)) + 2^(phi*F(n-2)))), where phi is the golden ratio. - Vladimir Shevelev, Jun 24 2010, Jun 27 2010

For n >= 1, a(n+1) = ceiling(phi*a(n)), if n is even and a(n+1) = floor(phi*a(n)), if n is odd (phi = golden ratio). - Vladimir Shevelev, Jul 01 2010

a(n) = 2*a(n-2) + a(n-3), n > 2. - Gary Detlefs, Sep 08 2010

a(2^n) = Product_{i=0..n-1} A000032(2^i). - Vladimir Shevelev, Nov 28 2010

a(n)^2 - a(n-1)^2 = a(n+1)*a(n-2), see A121646.

a(n) = sqrt((-1)^k*(a(n+k)^2 - a(k)*a(2n+k))), for any k. - Gary Detlefs, Dec 03 2010

F(2*n) = F(n+2)^2 - F(n+1)^2 - 2*F(n)^2. - Richard R. Forberg, Jun 04 2011

(-1)^(n+1) = F(n)^2 + F(n)*F(1+n) - F(1+n)^2.

F(n) = -F(n+2)(-2 + (F(n+1))^4 + 2*(F(n+1)^3*F(n+2)) - (F(n+1)*F(n+2))^2 2*F(n+1)(F(n+2))^3 + (F(n+2))^4)- F(n+1). - Artur Jasinski, Nov 17 2011

F(n) = 1 + Sum_{x=1..n-2} F(x). - Joseph P. Shoulak, Feb 05 2012

F(n) = 4*F(n-2) - 2*F(n-3) - F(n-6). - Gary Detlefs, Apr 01 2012

F(n) = round(phi^(n+1)/(phi+2)). - Thomas Ordowski, Apr 20 2012

From Sergei N. Gladkovskii, Jun 03 2012: (Start)
G.f.: A(x) = x/(1-x-x^2) = G(0)/sqrt(5) where G(k) = 1 - ((-1)^k)*2^k/(a^k - b*x*a^k*2^k/(b*x*2^k - 2*((-1)^k)*c^k/G(k+1))) and a=3+sqrt(5), b=1+sqrt(5), c=3-sqrt(5); (continued fraction, 3rd kind, 3-step).
Let E(x) be the e.g.f., i.e.,
E(x) = 1*x + 1/2*x^2 + 1/3*x^3 + 1/8*x^4 + 1/24*x^5 + 1/90*x^6 + 13/5040*x^7 + ...; then
E(x) = G(0)/sqrt(5); G(k) = 1 -((-1)^k)*2^k/(a^k - b*x*a^k*2^k/(b*x*2^k - 2*((-1)^k)*(k+1)*c^k/G(k+1))), where a=3+sqrt(5), b=1+sqrt(5), c=3-sqrt(5); (continued fraction, 3rd kind, 3-step).
(End)

From Hieronymus Fischer, Nov 30 2012: (Start)
F(n) = 1 + Sum_{j_1=1..n-2} 1 + Sum_{j_1=1..n-2} Sum_{j_2=1..j_1-2} 1 + Sum_{j_1=1..n-2} Sum_{j_2=1..j_1-2} Sum_{j_3=1..j_2-2} 1 + ... + Sum_{j_1=1..n-2} Sum_{j_2=1..j_1-2} Sum_{j_3=1..j_2-2} ... Sum_{j_k=1..j_(k-1)-2} 1, where k = floor((n-1)/2).
Example: F(6) = 1 + Sum_{j=1..4} 1 + Sum_{j=1..4} Sum_{k=1..(j-2)} 1 + 0 = 1 + (1 + 1 + 1 + 1) + (1 + (1 + 1)) = 8.
F(n) = Sum_{j=0..k} S(j+1,n-2j), where k = floor((n-1)/2) and the S(j,n) are the n-th j-simplex sums: S(1,n) = 1 is the 1-simplex sum, S(2,n) = Sum_{k=1..n} S(1,k) = 1+1+...+1 = n is the 2-simplex sum, S(3,n) = Sum_{k=1..n} S(2,k) = 1+2+3+...+n is the 3-simplex sum (= triangular numbers = A000217), S(4,n) = Sum_{k=1..n} S(3,k) = 1+3+6+...+n(n+1)/2 is the 4-simplex sum (= tetrahedral numbers = A000292) and so on.
Since S(j,n) = binomial(n-2+j,j-1), the formula above equals the well-known binomial formula, essentially. (End)

G.f.: A(x) = x / (1 - x / (1 - x / (1 + x))). - Michael Somos, Jan 04 2013

Sum_{n >= 1} (-1)^(n-1)/(a(n)*a(n+1)) = 1/phi (phi=golden ratio). - Vladimir Shevelev, Feb 22 2013

From Vladimir Shevelev, Feb 24 2013: (Start)
(1) Expression a(n+1) via a(n): a(n+1) = (a(n) + sqrt(5*(a(n))^2 + 4*(-1)^n))/2;
(2) Sum_{k=1...n} (-1)^(k-1)/(a(k)*a(k+1)) = a(n)/a(n+1);
(3) a(n)/a(n+1) = 1/phi + r(n), where |r(n)| < 1/(a(n+1)*a(n+2)). (End)

F(n+1) = F(n)/2 + sqrt((-1)^n + 5*F(n)^2/4), n >= 0. F(n+1) = U_n(i/2)/i^n, (U:= Chebyshev polynomial of the 2nd kind). - Bill Gosper, Mar 04 2013

G.f.: -Q(0) where Q(k) = 1 - (1+x)/(1 - x/(x - 1/Q(k+1))); (continued fraction). - Sergei N. Gladkovskii, Mar 06 2013

G.f.: x-1-1/x + 1/x/Q(0), where Q(k) = 1 - (k+1)*x/(1 - x/(x - (k+1)/Q(k+1))); (continued fraction). - Sergei N. Gladkovskii, Apr 23 2013

G.f.: x*G(0), where G(k)= 1 + x*(1+x)/(1 - x*(1+x)/(x*(1+x) + 1/G(k+1))); (continued fraction). - Sergei N. Gladkovskii, Jul 08 2013

G.f.: x^2 - 1 + 2*x^2/(W(0)-2), where W(k) = 1 + 1/(1 - x*(k + x)/(x*(k+1 + x) + 1/W(k+1))); (continued fraction). - Sergei N. Gladkovskii, Aug 28 2013

G.f.: Q(0) -1, where Q(k) = 1 + x^2 + (k+2)*x -x*(k+1 + x)/Q(k+1); (continued fraction). - Sergei N. Gladkovskii, Oct 06 2013

Let b(n) = b(n-1) + b(n-2), with b(0) = 0, b(1) = phi. Then, for n >= 2, F(n) = floor(b(n-1)) if n is even, F(n) = ceiling(b(n-1)), if n is odd, with convergence. - Richard R. Forberg, Jan 19 2014

a(n) = Sum_{t1*g(1)+t2*g(2)+...+tn*g(n)=n} multinomial(t1+t2 +...+tn,t1,t2,...,tn), where g(k)=2*k-1. - Mircea Merca, Feb 27 2014

F(n) = round(sqrt(F(n-1)^2 + F(n)^2 + F(n+1)^2)/2), for n > 0. This rule appears to apply to any sequence of the form a(n) = a(n-1) + a(n-2), for any two values of a(0) and a(1), if n is sufficiently large. - Richard R. Forberg, Jul 27 2014

F(n) = round(2/(1/F(n) + 1/F(n+1) + 1/F(n+2)), for n > 0. This rule also appears to apply to any sequence of the form a(n) = a(n-1) + a(n-2), for any two values of a(0) and a(1), if n is sufficiently large. - Richard R. Forberg, Aug 03 2014

F(n) = round(1/(Sum_{j>=n+2} 1/F(j))). - Richard R. Forberg, Aug 14 2014

a(n) = hypergeometric([-n/2+1/2, -n/2+1], [-n+1], -4) for n >= 2. - Peter Luschny, Sep 19 2014

F(n) = (L(n+1)^2 - L(n-1)^2)/(5*L(n)), where L(n) is A000032(n), with a similar inverse relationship. - Richard R. Forberg, Nov 17 2014

Consider the graph G[1-vertex;1-loop,2-loop] in comment above. Construct the power matrix array T(n,j) = [A^*j]*[S^*(j-1)] where A=(1,1,0,...) and S=(0,1,0,...)(A063524). [* is convolution operation] Define S^*0=I with I=(1,0,...). Then T(n,j) counts n-walks containing (j) loops and a(n-1) = Sum_{j=1...n} T(n,j). - David Neil McGrath, Nov 21 2014

Define F(-n) to be F(n) for n odd and -F(n) for n even. Then for all n and k, F(n) = F(k)*F(n-k+3) - F(k-1)*F(n-k+2) - F(k-2)*F(n-k) + (-1)^k*F(n-2k+2). - Charlie Marion, Dec 04 2014

F(n+k)^2 - L(k)*F(n)*F(n+k) + (-1)^k*F(n)^2 = (-1)^n*F(k)^2, if L(k) = A000032(k). - Alexander Samokrutov, Jul 20 2015

F(2*n) = F(n+1)^2 - F(n-1)^2, similar to Koshy (D) and Forberg 2011, but different. - Hermann Stamm-Wilbrandt, Aug 12 2015

F(n+1) = ceiling((1/phi)*Sum_{k=0..n} F(k)). - Tom Edgar, Sep 10 2015

a(n) = (L(n-3) + L(n+3))/10 where L(n)=A000032(n). - J. M. Bergot, Nov 25 2015

From Bob Selcoe, Mar 27 2016: (Start)
F(n) = (F(2n+k+1) - F(n+1)*F(n+k+1))/F(n+k), k >= 0.
Thus when k=0: F(n) = sqrt(F(2n+1) - F(n+1)^2).
F(n) = (F(3n) - F(n+1)^3 + F(n-1)^3)^(1/3).
F(n+2k) = binomial transform of any subsequence starting with F(n). Example F(6)=8: 1*8 = F(6)=8; 1*8 + 1*13 = F(8)=21; 1*8 + 2*13 + 1*21 = F(10)=55; 1*8 + 3*13 + 3*21 + 1*34 = F(12)=144, etc. This formula applies to Fibonacci-type sequences with any two seed values for a(0) and a(1) (e.g., Lucas sequence A000032: a(0)=2, a(1)=1).
(End)

F(n) = L(k)*F(n-k) + (-1)^(k+1)*F(n-2k) for all k >= 0, where L(k) = A000032(k). - Anton Zakharov, Aug 02 2016

From Ilya Gutkovskiy, Aug 03 2016: (Start)
a(n) = F_n(1), where F_n(x) are the Fibonacci polynomials.
Inverse binomial transform of A001906.
Number of zeros in substitution system {0 -> 11, 1 -> 1010} at step n from initial string "1" (1 -> 1010 -> 101011101011 -> ...) multiplied by 1/A000079(n). (End)

For n>=2, a(n) = 2^(n^2+n) - (4^n-2^n-1)*floor(2^(n^2+n)/(4^n-2^n-1)) - 2^n*floor(2^(n^2) - (2^n-1-1/2^n)*floor(2^(n^2+n)/(4^n-2^n-1))). - Benoit Cloitre, Apr 17 2017

For n > 0, a(n) = b(n+1) where b(n) = Sum_{k=1..n} b(n-k)*A000931(k-1), b(0) = 1. - J. Conrad, Apr 19 2017

f(n+1) = Sum_{j=0..floor(n/2)} Sum_{k=0..j} binomial(n-2j,k)*binomial(j,k). - Tony Foster III, Sep 04 2017

F(n) = Sum_{k=0..floor((n-1)/2)} ((n-k-1)! / ((n-2k-1)! * k!)). - Zhandos Mambetaliyev, Nov 08 2017

For x even, F(n) = (F(n+x) + F(n-x))/L(x). For x odd, F(n) = (F(n+x) - F(n-x))/L(x) where n >= x in both cases. Therefore F(n) = F(2*n)/L(n) for n >= 0. - David James Sycamore, May 04 2018

From Isaac Saffold, Jul 19 2018: (Start)
Let [a/p] denote the Legendre symbol. Then, for an odd prime p:
F(p+n) == [5/p]*F([5/p]+n) (mod p), if [5/p] = 1 or -1.
F(p+n) == 3*F(n) (mod p), if [5/p] = 0 (i.e., p = 5).
This is true for negative-indexed terms as well, if this sequence is extended by the negafibonacci numbers (i.e., F(-n) = A039834(n)). (End)

a(n) = A094718(4, n). a(n) = A101220(0, j, n).

a(n) = A090888(0, n+1) = A118654(0, n+1) = A118654(1, n-1) = A109754(0, n) = A109754(1, n-1), for n > 0.

a(n) = (L(n-3) + L(n-2) + L(n-1) + L(n))/5 with L(n)=A000032(n). - Art Baker, Jan 04 2019

F(n) = F(k-1)*F(abs(n-k-2)) + F(k-1)*F(n-k-1) + F(k)*F(abs(n-k-2)) + 2*F(k)*F(n-k-1), for n > k > 0. - Joseph M. Shunia, Aug 12 2019

F(n) = F(n-k+2)*F(k-1) + F(n-k+1)*F(k-2) for all k such that 2 <= k <= n. - Michael Tulskikh, Oct 09 2019

F(n)^2 - F(n+k)*F(n-k) = (-1)^(n+k) * F(k)^2 for 2 <= k <= n [Catalan's identity]. - Hermann Stamm-Wilbrandt, May 07 2021

Sum_{n>=1} 1/a(n) = A079586 is the reciprocal Fibonacci constant. - Gennady Eremin, Aug 06 2021

EXA For x = 0,1,2,3,4, x=1/(x+1) = 1, 1/2, 2/3, 3/5, 5/8. These fractions have numerators 1,1,2,3,5, which are the 2nd to 6th entries in the sequence. - Cino Hilliard, Sep 15 2008

MPLE

From Joerg Arndt, May 21 2013: (Start)
There are a(7)=13 compositions of 7 where there is a drop between every second pair of parts, starting with the first and second part:

```
01:  [ 2 1 2 1 1 ]
02:  [ 2 1 3 1 ]
03:  [ 2 1 4 ]
04:  [ 3 1 2 1 ]
05:  [ 3 1 3 ]
06:  [ 3 2 2 ]
07:  [ 4 1 2 ]
08:  [ 4 2 1 ]
09:  [ 4 3 ]
10:  [ 5 1 1 ]
11:  [ 5 2 ]
12:  [ 6 1 ]
13:  [ 7 ]
```

There are abs(a(6+1))=13 compositions of 6 where there is no rise between every second pair of parts, starting with the second and third part:

```
01:  [ 1 2 1 2 ]
02:  [ 1 3 1 1 ]
03:  [ 1 3 2 ]
04:  [ 1 4 1 ]
05:  [ 1 5 ]
06:  [ 2 2 1 1 ]
07:  [ 2 3 1 ]
08:  [ 2 4 ]
09:  [ 3 2 1 ]
10:  [ 3 3 ]
11:  [ 4 2 ]
12:  [ 5 1 ]
13:  [ 6 ]
```

(End)

Partially ordered partitions of (n-1) into parts 1,2,3 where only the order of the adjacent 1's and 2's are unimportant. E.g., a(8)=21. These are (331),(313),(133),(322),(232),(223),(3211),(2311),(1321),(2131),(1132),(2113),(31111),(13111),(11311),(11131),(11113),(2221),(22111),(211111),(1111111). - David Neil McGrath, Jul 25 2015

Consider the partitions of 7 with summands initially listed in nonincreasing order. Keep the 1's frozen in position,(indicated by "[]") and then allow the other summands to otherwise vary their order: 7; 6,[1]; 5,2; 2,5; 4,3; 3,4; 5,[1,1], 4,2,[1]; 2,4,[1]; 3,3,[1]; 3,3,2; 3,2,3; 2,3,3; 4,[1,1,1]; 3,2,[1,1]; 2,3,[1,1]; 2,2,2,[1]; 3,[1,1,1,1]; 2,2,[1,1,1]; 2,[1,1,1,1,1]; [1,1,1,1,1,1,1]. There are 21 = a(7+1) arrangements in all. - Gregory L. Simay, Jun 14 2016

MAPLE

```
A000045 := proc(n) combinat[fibonacci](n); end;
ZL:=[S, {a = Atom, b = Atom, S = Prod(X, Sequence(Prod(X, b))), X = Sequence(b, card >= 1)}, unlabelled]: seq(combstruct[count](ZL, size=n), n=0..38); # Zerinvary Lajos, Apr 04 2008
spec := [B, {B=Sequence(Set(Z, card>1))}, unlabeled ]: seq(combstruct[count](spec, size=n), n=1..39); # Zerinvary Lajos, Apr 04 2008
# The following Maple command isFib(n) yields true or false depending on whether n is a Fibonacci number or not.
with(combinat): isFib := proc(n) local a: a := proc(n) local j: for j while fibonacci(j) <= n do fibonacci(j) end do: fibonacci(j-1) end proc: evalb(a(n) = n) end proc: # Emeric Deutsch, Nov 11 2014
```

MATHEMATICA

```
Table[Fibonacci[k], {k, 0, 50}] (* Mohammad K. Azarian, Jul 11 2015 *)
Table[2^n Sqrt @ Product[(Cos[Pi k/(n + 1)]^2 + 1/4), {k, n}] // FullSimplify, {n, 15}]; (* Kasteleyn's formula specialized, Sarah-Marie Belcastro (smbelcas(AT)toroidalsnark.net), Jul 04 2009 *)
LinearRecurrence[{1, 1}, {0, 1}, 40] (* Harvey P. Dale, Aug 03 2014 *)
Fibonacci[Range[0, 20]] (* Eric W. Weisstein, Sep 22 2017 *)
CoefficientList[Series[-(x/(-1 + x + x^2)), {x, 0, 20}], x] (* Eric W. Weisstein, Sep 22 2017 *)
```

PROG

```
(Axiom) [fibonacci(n) for n in 0..50]
(MAGMA) [Fibonacci(n): n in [0..38]];
(MAGMA) [0, 1] cat [n: n in [1..50000000] | IsSquare(5*n^2-4) or IsSquare(5*n^2+4)]; // Vincenzo Librandi, Nov 19 2014
(Maxima) makelist(fib(n), n, 0, 100); /* Martin Ettl, Oct 21 2012 */
(PARI) a(n) = fibonacci(n)
(PARI) a(n) = imag(quadgen(5)^n)
(PARI) a(n)=my(phi=quadgen(5)); (phi^n-(-1/phi)^n)/(2*phi-1) \\ Charles R Greathouse IV, Jun 17 2012
(PARI) a(n)=polcoeff(sum(m=0, n, x^m*prod(k=1, m, k+x +x*O(x^n))/prod(k=1, m, 1+k*x +x*O(x^n))), n) \\ Paul D. Hanna, Oct 26 2013
(Python) # From Jaap Spies, Jan 05 2007:
def fib():
    """ Generates the Fibonacci numbers, starting with 0 """
    x, y = 0, 1
    while 1:
        yield x
        x, y = y, x+y
f = fib()
a = [next(f) for _ in range(100)]
def A000045(n):
    """ Returns Fibonacci number with index n, offset 0 """
    return a[n]
def A000045_list(N):
    """ Returns a list of the first n Fibonacci numbers """
    return a[:N]
(Python) # As b-file:
from gmpy2 import fib
for n in range(100): print(str(n) + " " + str(fib(n)))  # Bruno Berselli, Dec 06 2016
(Sage) # Demonstration program from Jaap Spies:
a = sloane.A000045; # choose sequence
print(a)            # This returns the name of the sequence.
print(a(38))        # This returns the 38th number of the sequence.
print(a.list(39))   # This returns a list of the first 39 numbers.
(Sage) # Alternatively:
a = BinaryRecurrenceSequence(1, 1); print([a(n) for n in range(20)])
# Closed form integer formula with F(1) = 0 from Paul Hankin (use only for fun).
F = lambda n: (4<<(n-1)*(n+2))//((4<<2*(n-1))-(2<<(n-1))-1)&((2<<(n-1))-1)
print([F(n) for n in range(20)]) # Peter Luschny, Aug 28 2016
(Sage) print(list(fibonacci_sequence(0, 40))) # Bruno Berselli, Jun 26 2014
(Haskell)
-- Based on code from http://www.haskell.org/haskellwiki/The_Fibonacci_sequence
-- which also has other versions.
fib :: Int -> Integer
fib n = fibs !! n
    where
        fibs = 0 : 1 : zipWith (+) fibs (tail fibs)
{- Example of use: map fib [0..38] Gerald McGarvey, Sep 29 2009 -}
(Julia)
function fib(n)
   F = BigInt[1 1; 1 0]
   Fn = F^n
   Fn[2, 1]
end
println([fib(n) for n in 0:38]) # Peter Luschny, Feb 23 2017
(GAP)
Fib:=[0, 1];; for n in [3..10^3] do Fib[n]:=Fib[n-1]+Fib[n-2]; od; Fib;  #  Muniru A Asiru, Sep 03 2017
(Scheme)
;; The following definition uses macro definec for the memoization (caching) of the results. See http://oeis.org/wiki/Memoization#Scheme
(definec (A000045 n) (if (< n 2) n (+ (A000045 (- n 1)) (A000045 (- n 2))))) ;; Antti Karttunen, Oct 06 2017
(Scala) def fibonacci(n: BigInt): BigInt = {
  val zero = BigInt(0)
  def fibTail(n: BigInt, a: BigInt, b: BigInt): BigInt = n match {
    case `zero` => a
    case _ => fibTail(n - 1, b, a + b)
```

```
  }
  fibTail(n, 0, 1)
} // Based on "Case 3: Tail Recursion" from Carrasquel (2016) link
(0 to 49).map(fibonacci(_)) // Alonso del Arte, Apr 13 2019
```

CROSSREFS

Cf. A039834 (signed Fibonacci numbers), A001519 (F(2n-1)), A001906 (F(2n)), A001690 (complement), A000213, A000288, A000322, A000383, A060455, A030186, A020695, A020701, A071679, A099731, A100492, A094216, A094638, A000108, A101399, A101400, A001611, A000071, A157725, A001911, A157726, A006327, A157727, A157728, A157729, A167616, A059929, A144152, A152063, A114690, A003893, A000032, A060441, A000930, A003269, A000957, A057078, A007317, A091867, A104597, A249548, A262342, A001060, A022095, A072649, A163733, A073133, A166861 (Euler Transform), A337009 (Multiset Transf.).
First row of arrays A103323, A172236, A234357. Second row of arrays A099390, A048887, and A092921 (k-generalized Fibonacci numbers).
Cf. A001175 (Pisano periods), A001177 (Entry points), A001176 (number of zeros in a fundamental period).
Fibonacci-Pascal triangles: A027926, A036355, A037027, A074829, A105809, A109906, A111006, A114197, A162741, A228074.
Fibonacci-Cayley triangle: A327992.
Boustrophedon transforms: A000738, A000744.
Powers: A103323, A105317, A254719.
Numbers of prime factors: A022307 and A038575.

KEYWORD

nonn,core,nice,easy,hear,changed

AUTHOR

N. J. A. Sloane, 1964

STATUS

approved

A000041 a(n) is the number of partitions of n (the partition numbers). (Formerly M0663 N0244)

+30
2953

1, 1, 2, 3, 5, 7, 11, 15, 22, 30, 42, 56, 77, 101, 135, 176, 231, 297, 385, 490, 627, 792, 1002, 1255, 1575, 1958, 2436, 3010, 3718, 4565, 5604, 6842, 8349, 10143, 12310, 14883, 17977, 21637, 26015, 31185, 37338, 44583, 53174, 63261, 75175, 89134, 105558, 124754, 147273, 173525 (list; graph; refs; listen; history; edit; text; internal format)

OFFSET

0,3

COMMENTS

Also number of nonnegative solutions to b + 2c + 3d + 4e + ... = n and the number of nonnegative solutions to 2c + 3d + 4e + ... <= n. - Henry Bottomley, Apr 17 2001
a(n) is also the number of conjugacy classes in the symmetric group S_n (and the number of irreducible representations of S_n).
Also the number of rooted trees with n+1 nodes and height at most 2.
Coincides with the sequence of numbers of nilpotent conjugacy classes in the Lie algebras gl(n). A006950, A015128 and this sequence together cover the nilpotent conjugacy classes in the classical A,B,**C**,D series of Lie algebras. - Alexander Elashvili, Sep 08 2003
Number of distinct Abelian groups of order p^n, where p is prime (the number is independent of p). - Lekraj Beedassy, Oct 16 2004
Number of graphs on n vertices that do not contain P3 as an induced subgraph. - Washington Bomfim, May 10 2005
Numbers of terms to be added when expanding the n-th derivative of 1/f(x). - Thomas Baruchel, Nov 07 2005
Sequence agrees with expansion of Molien series for symmetric group S_n up to the term in x^n. - Maurice D. Craig (towenaar(AT)optusnet.com.au), Oct 30 2006
Also the number of nonnegative integer solutions to x_1 + x_2 + x_3 + ... + x_n = n such that n >= x_1 >= x_2 >= x_3 >= ... >= x_n >= 0, because by letting y_k = x_k - x_(k+1) >= 0 (where 0 < k < n) we get y_1 + 2y_2 + 3y_3 + ... + (n-1)y_(n-1) + nx_n = n. - Werner Grundlingh (wgrundlingh(AT)gmail.com), Mar 14 2007
Let P(z) := Sum_{j>=0} b_j z^j, b_0 != 0. Then 1/P(z) = Sum_{j>=0} c_j z^j, where the c_j must be computed from the infinite triangular system b_0 c_0 = 1, b_0 c_1 + b_1 c_0 = 0 and so on (Cauchy products of the coefficients set to zero). The n-th partition number arises as the number of terms in the numerator of the expression for c_n: The coefficient c_n of the inverted power series is a fraction with b_0^(n+1) in the denominator and in its numerator having a(n) products of n coefficients b_i each. The partitions may be read off from the indices of the b_i. - Peter **C**. Heinig (algorithms(AT)gmx.de), Apr 09 2007
A026820(a(n),n) = A134737(n) for n > 0. - Reinhard Zumkeller, Nov 07 2007
Equals row sums of triangle A137683. - Gary W. Adamson, Feb 05 2008
a(n) is the number of different ways to run up a staircase with n steps, taking steps of sizes 1, 2, 3, ... and r (r <= n), where the order is not important and there is no restriction on the number or the size of each step taken. - Mohammad K. Azarian, May 21 2008
Equals the eigenvector of triangle A145006 and row sums of the eigentriangle of the partition numbers, A145007. - Gary W. Adamson, Sep 28 2008
Starting with offset 1 = INVERT transform of (1, 1, 0, 0, -1, 0, -1, ...), where A080995, the characteristic function of A001318 (1, 2, 5, 7, 12, ...) is signed (++ -- ++, ...) as to 1's. This is equivalent to lim_{n=1..inf} A145006^n as a vector. The INVERT transform of (1, 1, 0, 0, -1, ...) begins (1, 2,..) then for each successive operation we take a dot product of (1, 1, 0, 0, -1, ...) in reverse and the ongoing results of our series (1, 2, 3, 5, 7, ...) then add the result to the next term in (1, 1, 0, 0, -1, ...). For example, a(7) = 15 = (0, -1, 0, 0, 1, 1) dot (1, 2, 3, 5, 7, 11) = (0*1, (-1)*2, 0*3, 0*5, 1*7, 1*11) = (-2 + 7 + 11) = 16, then add to (-1) = 15. - Gary W. Adamson, Oct 05 2008
Convolved with A147843 = A000203 prefaced with a zero: (0, 1, 3, 4, 7, ...). - Gary W. Adamson, Nov 15 2008
Equals an infinite convolution product_(1, 1, 1, ...)*(1, 0, 1, 0, 1, ...)*(1, 0, 0, 1, 0, 0, 1, ...)*(1, 0, 0, 0, 1, 0, 0, 0, 1, ...)*...; = a*b*c*...; where a = (1/(1-x)), b = (1/(1-x^2)), c = (1/(1-x^3)), etc. An array by rows: row 1 = a, row 2 = a*b, row 3 = a*b*c, ...; gives:

```
1, 1, 1, 1, 1, 1,  1,  1,  1,  1, ... = (a)
1, 1, 2, 2, 3, 3,  4,  4,  5,  5, ... = (a*b)
1, 1, 2, 3, 4, 5,  7,  8, 10, 11, ... = (a*b*c)
1, 1, 2, 3, 4, 5,  6,  9, 11, 17, ... = (a*b*c*d)
1, 1, 2, 3, 5, 5,  7, 10, 13, 18, ... = (a*b*c*d*e)
1, 1, 2, 3, 5, 7, 11, 14, 20, 25, ... = (a*b*c*d*e*f)
1, 1, 2, 3, 5, 7, 11, 15, 21, 27, ... = (a*b*c*d*e*f*g)
1, 1, 2, 3, 5, 7, 11, 15, 22, 28, ... = (a*b*c*d*e*f*g*h)
1, 1, 2, 3, 5, 7, 11, 15, 22, 29, ... = (a*b*c*d*e*f*g*h*i)
```

... with rows tending to A000041. Partition triangles A058398 = ascending antidiagonals. Partition triangle A008284 reversal of A058398. - Gary W. Adamson, Jun 12 2009
Starting with offset 1 = row sums of triangle A168532. - Gary W. Adamson, Nov 28 2009
P(x) = A(x)/A(x^2) with P(x) = (1+x+2x^2+3x^3+5x^4+7x^5 + ...),
and A(x) = (1 + x + 3x^2 + 4x^3 + 10x^4 + 13x^5 + ...),
and A(x^2) = (1 + x^2 + 3x^4 + 4x^6 + 10x^8 + ...), where A092119 = (1, 1, 3, 4, 10, ...) = Euler transform of the ruler sequence, A001511. - Gary W. Adamson, Feb 11 2010
Equals row sums of triangle A173304. - Gary W. Adamson, Feb 15 2010
p(x) = A(x)*A(x^2), A(x) = A174065; p(x) = B(x)*B(x^3), B(x) = A174068. Equals row sums of triangles A174066 and A174067. - Gary W. Adamson, Mar 06 2010
Triangle A113685 is equivalent to p(x) = p(x^2) * A000009(x). Triangle A176202 is equivalent to p(x) = p(x^3) * A000726(x). - Gary W. Adamson, Apr 11 2010
A sequence of positive integers p = p_1 ... p_k is a descending partition of the positive integer n if p_1 + ... + p_k = n and p_1 >= ... >= p_k. If formally needed p_j = 0 is appended to p for j > k. Let P_n denote the set of these partition for some n >= 1. Then a(n) = 1 + Sum_{p in P_n} floor((p_1-1)/(p_2+1)). (Cf. A000065, where the formula reduces to the sum.) Proof in Kelleher and O'Sullivan (2009). For example a(6) = 1 + 0 + 0 + 0 + 0 + 1 + 0 + 0 + 1 + 1 + 2 + 5 = 11. - Peter Luschny, Oct 24 2010
Let n = Sum(k_(p_m) p_m) = k_1 + 2k_2 + 5k_5 + 7k_7 + ..., where p_m is the m-th generalized pentagonal number (A001318). Then a(n) is the sum over all such pentagonal partitions of n of (-1)^(k_5+k_7 + k_22 + ...) (k_1 + k_2 + k_5 + ...)! /(k_1! k_2! k_5! ...), where the exponent of (-1) is the sum of all the k's corresponding to even-indexed GPN's. - Jerome Malenfant, Feb 14 2011
The matrix of a(n) values

```
a(0)
a(1) a(0)
a(2) a(1) a(0)
a(3) a(2) a(1) a(0)
....
a(n) a(n-1) a(n-2) ... a(0)
```

is the inverse of the matrix

```
  1
 -1  1
 -1 -1  1
  0 -1 -1  1
 ....
 -d_n  -d_(n-1) -d_(n-2) ... -d_1  1
```
where d_q = (-1)^(m+1) if q = m(3m-1)/2 = the m-th generalized pentagonal number (A001318), = 0 otherwise. - Jerome Malenfant, Feb 14 2011
Equals row sums of triangle A187566. - Gary W. Adamson, Mar 21 2011
Let k > 0 be an integer, and let i_1, i_2, ..., i_k be distinct integers such that 1 <= i_1 < i_2 < ... < i_k. Then, equivalently, a(n) equals the number of partitions of N = n + i_1 + i_2 + ... + i_k in which each i_j (1 <= j <= k) appears as a part at least once. To see this, note that the partitions of N of this class must be in 1-to-1 correspondence with the partitions of n, since N - i_1 - i_2 - ... - i_k = n. - L. Edson Jeffery, Apr 16 2011
a(n) is the number of distinct degree sequences over all free trees having n + 2 nodes. Take a partition of the integer n, add 1 to each part and append as many 1's as needed so that the total is 2n + 2. Now we have a degree sequence of a tree with n + 2 nodes. Example: The partition 3 + 2 + 1 = 6 corresponds to the degree sequence {4, 3, 2, 1, 1, 1, 1, 1} of a tree with 8 vertices. - Geoffrey Critzer, Apr 16 2011
a(n) is number of distinct characteristic polynomials among n! of permutations matrices size n X n. - Artur Jasinski, Oct 24 2011
Conjecture: starting with offset 1 represents the numbers of ordered compositions of n using the signed (++--++...) terms of A001318 starting (1, 2, -5, -7, 12, 15, ...). - Gary W. Adamson, Apr 04 2013 (this is true by the pentagonal number theorem, Joerg Arndt, Apr 08 2013)
a(n) is also number of terms in expansion of the n-th derivative of log(f(x)). In Mathematica notation: Table[Length[Together[f[x]^n * D[Log[f[x]], {x, n}]]], {n, 1, 20}]. - Vaclav Kotesovec, Jun 21 2013
Conjecture: No a(n) has the form x^m with m > 1 and x > 1. - Zhi-Wei Sun, Dec 02 2013
Partitions of n that contain a part p are the partitions of n - p. Thus, number of partitions of m*n - r that include k*n as a part is A000041(h*n-r), where h = m - k >= 0, n >= 2, 0 <= r < n; see A111295 as an example. - Clark Kimberling, Mar 03 2014
a(n) is the number of compositions of n into positive parts avoiding the pattern [1, 2]. - Bob Selcoe, Jul 08 2014
Conjecture: For any j there exists k such that all primes p <= A000040(j) are factors of one or more a(n) <= a(k). Growth of this coverage is slow and irregular. k = 1067 covers the first 102 primes, thus slower than A000027. - Richard R. Forberg, Dec 08 2014
a(n) is the number of nilpotent conjugacy classes in the order-preserving, order-decreasing and (order-preserving and order-decreasing) injective transformation semigroups. - Ugbene Ifeanyichukwu, Jun 03 2015
Define a segmented partition a(n,k, <s(1)..s(j)>) to be a partition of n with exactly k parts, with s(j) parts t(j) identical to each other and distinct from all the other parts. Note that n >= k, j <= k, 0 <= s(j) <= k, s(1)t(1) + ... + s(j)t(j) = n and s(1) + ... + s(j) = k. Then there are up to a(k) segmented partitions of n with exactly k parts. - Gregory L. Simay, Nov 08 2015
(End)
From Gregory L. Simay, Nov 09 2015: (Start)
The polynomials for a(n, k, <s(1), ..., s(j)>) have degree j-1.
  a(n, k, <k>) = 1 if n = 0 mod k, = 0 otherwise
  a(rn, rk, <r*s(1), ..., r*s(j)>) = a(n, k, <s(1), ..., s(j)>)
  a(n odd, k, <all s(j) even>) = 0
Established results can be recast in terms of segmented partitions:
  For j(j+1)/2 <= n < (j+1)(j+2)/2, A000009(n) = a(n, 1, <1>) + ... + a(n, j, <j 1's>), j < n
  a(n, k, <j 1's> = a(n - j(j-1)/2, k)
(End)
a(10^20) was computed using the NIST Arb package. It has 11140086260 digits and its head and tail sections are 18381765...88091448. See the Johansson 2015 link. - Stanislav Sykora, Feb 01 2016
Satisfies Benford's law [**Anderson**-Rolen-Stoehr, 2011]. - N. J. A. Sloane, Feb 08 2017
The partition function p(n) is log-concave for all n>25 [DeSalvo-Pak, 2014]. - Michel Marcus, Apr 30 2019
a(n) is also the dimension of the n-th cohomology of the infinite real Grassmannian with coefficients in Z/2. - Luuk Stehouwer, Jun 06 2021

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Index entries for sequences related to rooted trees
Index entries for sequences related to Benford's law

FORMULA

G.f.: Product_{k>0} 1/(1-x^k) = Sum_{k>= 0} x^k Product_{i = 1..k} 1/(1-x^i) = 1 + Sum_{k>0} x^(k^2)/(Product_{i = 1..k} (1-x^i))^2.
G.f.: 1 + Sum_{n>=1} x^n/(Product_{k>=n} 1-x^k). - Joerg Arndt, Jan 29 2011
a(n) - a(n-1) - a(n-2) + a(n-5) + a(n-7) - a(n-12) - a(n-15) + ... = 0, where the sum is over n-k and k is a generalized pentagonal number (A001318) <= n and the sign of the k-th term is (-1)^([(k+1)/2]). See A001318 for a good way to remember this!
a(n) = (1/n) * Sum_{k=0..n-1} sigma(n-k)*a(k), where sigma(k) is the sum of divisors of k (A000203).
a(n) ~ 1/(4*n*sqrt(3)) * e^(Pi * sqrt(2n/3)) as n -> infinity (Hardy and Ramanujan). See A050811.
a(n) = a(0)*b(n) + a(1)*b(n-2) + a(2)*b(n-4) + ... where b = A000009.
From Jon E. Schoenfield, Aug 17 2014: (Start)
It appears that the above approximation from Hardy and Ramanujan can be refined as
a(n) ~ 1/(4*n*sqrt(3)) * e^(Pi * sqrt(2n/3 + c0 + c1/n^(1/2) + c2/n + c3/n^(3/2) + c4/n^2 + ...)), where the coefficients c0 through c4 are approximately

```
 c0 = -0.230420145062453320665537
 c1 = -0.0178416569128570889793
 c2 =  0.0051329911273
 c3 = -0.0011129404
 c4 =  0.0009573,
```

as n -> infinity. (End)
From Vaclav Kotesovec, May 29 2016 (c4 added Nov 07 2016): (Start)

```
c0 = -0.230420145062453320665536704197233... = -1/36 - 2/Pi^2
c1 = -0.0178416569128570889795021353499 49... = 1/(6*sqrt(6)*Pi) - sqrt(3/2)/Pi^3
c2 =  0.00513299112734216759457639163355 9... = 1/(2*Pi^4)
c3 = -0.00111294048955976090823660284349 7... = 3*sqrt(3/2)/(4*Pi^5) - 5/(16*sqrt(6)*Pi^3)
c4 =  0.000957343284806972958968694349196... = 1/(576*Pi^2) - 1/(24*Pi^4) + 93/(80*Pi^6)
```

a(n) ~ exp(Pi*sqrt(2*n/3))/(4*sqrt(3)*n) * (1 - (sqrt(3/2)/Pi + Pi/(24*sqrt(6)))/sqrt(n) + (1/16 + Pi^2/6912)/n).
a(n) ~ exp(Pi*sqrt(2*n/3) - (sqrt(3/2)/Pi + Pi/(24*sqrt(6)))/sqrt(n) + (1/24 - 3/(4*Pi^2))/n) / (4*sqrt(3)*n).
(End)
a(n) < exp((2/3)^(1/2) Pi sqrt(n)) (Ayoub, p. 197).
G.f.: Product (1+x^m)^A001511(m); m=1..inf. - Vladeta Jovovic, Mar 26 2004
a(n) = Sum_{i=0..n-1} P(i, n-i), where P(x, y) is the number of partitions of x into at most y parts and P(0, y)=1. - Jon Perry, Jun 16 2003
G.f.: Product_{i>=1} Product_{j>=0} (1+x^((2i-1)*2^j))^(j+1). - Jon Perry, Jun 06 2004
G.f. e^(Sum_{k>0} (x^k/(1-x^k)/k)). - Franklin T. Adams-Watters, Feb 08 2006
a(n) = A114099(9*n). - Reinhard Zumkeller, Feb 15 2006
Euler transform of all 1's sequence (A000012). Weighout transform of A001511. - Franklin T. Adams-Watters, Mar 15 2006
a(n) = A027187(n) + A027193(n) = A000701(n) + A046682(n). - Reinhard Zumkeller, Apr 22 2006
Convolved with A152537 gives A000079, powers of 2. - Gary W. Adamson, Dec 06 2008
a(n) = A026820(n, n); a(n) = A108949(n) + A045931(n) + A108950(n) = A130780(n) + A171966(n) - A045931(n) = A045931(n) + A171967(n). - Reinhard Zumkeller, Jan 21 2010
a(n) = Tr(n)/(24*n-1) = A183011(n)/A183010(n), n>=1. See the Bruinier-Ono paper in the Links. - Omar E. Pol, Jan 23 2011
From Jerome Malenfant, Feb 14 2011: (Start)
a(n) = determinant of the n X n Toeplitz matrix:

```
   1  -1
   1   1  -1
   0   1   1  -1
   0   0   1   1  -1
  -1   0   0   1   1  -1
   . . .
  d_n  d_(n-1) d_(n-2)...1
```

where d_q = (-1)^(m+1) if q = m(3m-1)/2 = p_m, the m-th generalized pentagonal number (A001318), otherwise d_q = 0. Note that the 1's run along the diagonal and the -1's are on the superdiagonal. The (n-1) row (not written) would end with ... 1 -1. (End)
Empirical: let F*(x) = Sum_{n=0..infinity} p(n)*exp(-Pi*x*(n+1)), then F*(2/5) = 1/sqrt(5) to a precision of 13 digits.
F*(4/5) = 1/2+3/2/sqrt(5)-sqrt(1/2*(1+3/sqrt(5))) to a precision of 28 digits. These are the only values found for a/b when a/b is from F60, Farey fractions up to 60. The number for F*(4/5) is one of the real roots of 25*x^4 - 50*x^3 - 10*x^2 - 10*x + 1. Note here the exponent (n+1) compared to the standard notation with n starting at 0. - Simon Plouffe, Feb 23 2011
The constant (2^(7/8)*GAMMA(3/4))/(exp(Pi/6)*Pi^(1/4)) = 1.0000034873... when expanded in base exp(4*Pi) will give the first 52 terms of a(n), n>0, the precision needed is 300 decimal digits. - Simon Plouffe, Mar 02 2011
a(n) = A035363(2n). - Omar E. Pol, Nov 20 2009
G.f.: A(x)=1+x/(G(0)-x); G(k) = 1 + x - x^(k+1) - x*(1-x^(k+1))/G(k+1); (continued fraction Euler's kind, 1-step). - Sergei N. Gladkovskii, Jan 25 2012
Convolution of A010815 with A000712. - Gary W. Adamson, Jul 20 2012
G.f.: 1 + x*(1 - G(0))/(1-x) where G(k) = 1 - 1/(1-x^(k+1))/(1-x/(x-1/G(k+1))); (continued fraction). - Sergei N. Gladkovskii, Jan 22 2013
G.f.: Q(0) where Q(k) = 1 + x^(4*k+1)/((x^(2*k+1)-1)^2 - x^(4*k+3)*(x^(2*k+1)-1)^2/(x^(4*k+3) + (x^(2*k+2)-1)^2/Q(k+1))); (continued fraction). - Sergei N. Gladkovskii, Feb 16 2013
a(n) = 24*spt(n) + 12*N_2(n) - Tr(n) = 24*A092269(n) + 12*A220908(n) - A183011(n), n >= 1. - Omar E. Pol, Feb 17 2013
G.f.: 1/(x; x)_{inf} where (a; q)_k is the q-Pochhammer symbol. - Vladimir Reshetnikov, Apr 24 2013
a(n) = A066186(n)/n, n >= 1. - Omar E. Pol, Aug 16 2013
From Peter Bala, Dec 23 2013: (Start)
a(n-1) = Sum_{parts k in all partitions of n} mu(k), where mu(k) is the arithmetical Möbius function (see A008683).
Let P(2,n) denote the set of partitions of n into parts k >= 2. Then a(n-2) = -Sum_{parts k in all partitions in P(2,n)} mu(k).
n*(a(n) - a(n-1)) = Sum_{parts k in all partitions in P(2,n)} k (see A138880).
Let P(3,n) denote the set of partitions of n into parts k >= 3. Then
a(n-3) = (1/2)*Sum_{parts k in all partitions in P(3,n)} phi(k), where phi(k) is the Euler totient function (see A000010). Using this result and Merten's theorem on the average order of the phi function, we can find an approximate 3-term recurrence for the partition function:a(n) ~ a(n-1) + a(n-2) + (Pi^2/(3*n) - 1)*a(n-3). For example, substituting the values a(47) = 124,754, a(48) = 147,273 and a(49) = 173,525 into the recurrence gives the approximation a(50) ~ 204,252.48... compared with the true value a(50) = 204,226. (End)
a(n) = Sum_{k=1..n+1} (-1)^(n+1-k)*A000203(k)*A002040(n+1-k). - Mircea Merca, Feb 27 2014
a(n) = A240690(n) + A240690(n+1), n >= 1. - Omar E. Pol, Mar 16 2015
From Gary W. Adamson, Jun 22 2015: (Start)
A production matrix for the sequence with offset 1 is M, an infinite n x n matrix of the following form:

```
  a, 1, 0, 0, 0, 0, ...
  b, 0, 1, 0, 0, 0, ...
  c, 0, 0, 1, 0, 0, ...
  d, 0, 0, 0, 1, 0, ...
.
.
```

... such that (a, b, **c**, d, ...) is the signed version of A080995 with offset 1: (1,1,0,0,-1,0,-1,...)
and a(n) is the upper left term of M^n.
This operation is equivalent to the g.f. (1 + x + 2x^2 + 3x^3 + 5x^4 + ...) = 1/(1 - x - x^2 + x^5 + x^7 - x^12 - x^15 + x^22 + ...). (End)
G.f.: x^(1/24)/eta(log(x)/(2 Pi i)). - Thomas Baruchel, Jan 09 2016, after Michael Somos (after Richard Dedekind).
a(n) = Sum_{k=-inf..+inf} (-1)^k a(n-k(3k-1)/2) with a(0)=1 and a(negative)=0. The sum can be restricted to the (finite) range from k = (1-sqrt(1-24n))/6 to (1+sqrt(1-24n))/6), since all terms outside this range are zero. - Jos Koot, Jun 01 2016
G.f.: (conjecture) (r(x) * r(x^2) * r(x^4) * r(x^8) * ...) where r(x) is A000009: (1, 1, 1, 2, 2, 3, 4, ...). - Gary W. Adamson, Sep 18 2016; Doron Zeilberger observed today that "This follows immediately from Euler's formula 1/(1-z) = (1+z)*(1+z^2)*(1+z^4)*(1+z^8)*..." Gary W. Adamson, Sep 20 2016
a(n) ~ 2*Pi * BesselI(3/2, sqrt(24*n-1)*Pi/6) / (24*n-1)^(3/4). - Vaclav Kotesovec, Jan 11 2017
G.f.: Product_{k>=1} (1 + x^k)/(1 - x^(2*k)). - Ilya Gutkovskiy, Jan 23 2018
a(n) = p(1, n) where p(k, n) = p(k+1, n) + p(k, n-k) if k < n, 1 if k = n, and 0 if k > n. p(k, n) is the number of partitions of n into parts >= k. - Lorraine Lee, Jan 28 2020
Sum_{n>=1} 1/a(n) = A078506. - Amiram Eldar, Nov 01 2020
Sum_{n>=0} a(n)/2^n = A065446. - Amiram Eldar, Jan 19 2021
From Simon Plouffe, Mar 12 2021: (Start)
Empirical: Sum_{n>=0}, a(n)/exp(Pi*(n-1)) = 2^(3/8)*Gamma(3/4)/(Pi^(1/4)*exp(Pi/24)).
Empirical: Sum_{n>=0}, a(n)/exp(2*Pi*(n-1)) = 2^(1/2)*Gamma(3/4)/(Pi^(1/4)*exp(Pi/12)). (End) [These are the reciprocals of phi(exp(-Pi)) (A259148) and phi(exp(-2*Pi)) (A259149), where phi(q) is the Euler modular function. See B. **C**. Berndt (RLN, Vol. V, p. 326), and

formulas (13) and (14) in I. Mező, 2013. - Peter Luschny, Mar 13 2021]

EXAMPLE

a(5) = 7 because there are seven partitions of 5, namely: {1, 1, 1, 1, 1}, {2, 1, 1, 1}, {2, 2, 1}, {3, 1, 1}, {3, 2}, {4, 1}, {5}. - Bob Selcoe, Jul 08 2014
G.f. = 1 + x + 2*x^2 + 3*x^3 + 5*x^4 + 7*x^5 + 11*x^6 + 15*x^7 + 22*x^8 + ...
G.f. = 1/q + q^23 + 2*q^47 + 3*q^71 + 5*q^95 + 7*q^119 + 11*q^143 + 15*q^167 + ...
From Gregory L. Simay, Nov 08 2015: (Start)
There are up to a(4)=5 segmented partitions of the partitions of n with exactly 4 parts. They are a(n,4, <4>), a(n,4,<3,1>), a(n,4,<2,2>), a(n,4,<2,1,1>), a(n,4,<1,1,1,1>).
The partition 8,8,8,8 is counted in a(32,4,<4>).
The partition 9,9,9,5 is counted in a(32,4,<3,1>).
The partition 11,11,5,5 is counted in a(32,4,<2,2>).
The partition 13,13,5,1 is counted in a(32,4,<2,1,1>).
The partition 14,9,6,3 is counted in a(32,4,<1,1,1,1>).
a(n odd,4,<2,2>) = 0.
a(12, 6, <2,2,2>) = a(6,3,<1,1,1>) = a(6-3,3) = a(3,3) = 1. The lone partition is 3,3,2,2,1,1.
(End)

MAPLE

```
A000041 := n -> combinat:-numbpart(n): [seq(A000041(n), n=0..50)]; # Warning: Maple 10 and 11 give incorrect answers in some cases: A110375.
spec := [B, {B=Set(Set(Z, card>=1))}, unlabeled ];
[seq(combstruct[count](spec, size=n), n=0..50)];
with(combstruct):ZL0:=[S, {S=Set(Cycle(Z, card>0))}, unlabeled]: seq(count(ZL0, size=n), n=0..45); # Zerinvary Lajos, Sep 24 2007
G:={P=Set(Set(Atom, card>0))}: combstruct[gfsolve](G, labeled, x); seq(combstruct[count]([P, G, unlabeled], size=i), i=0..45); # Zerinvary Lajos, Dec 16 2007
# Using the function EULER from Transforms (see link at the bottom of the page).
1, op(EULER([seq(1, n=1..49)])); # Peter Luschny, Aug 19 2020
```

MATHEMATICA

```
Table[ PartitionsP[n], {n, 0, 45}]
a[ n_] := SeriesCoefficient[ q^(1/24) / DedekindEta[ Log[q] / (2 Pi I)], {q, 0, n}]; (* Michael Somos, Jul 11 2011 *)
a[ n_] := SeriesCoefficient[ 1 / Product[ 1 - x^k, {k, n}], {x, 0, n}]; (* Michael Somos, Jul 11 2011 *)
CoefficientList[1/QPochhammer[q] + O[q]^100, q] (* Jean-François Alcover, Nov 25 2015 *)
```

PROG

```
(MAGMA) a:= func< n | NumberOfPartitions(n) >; [ a(n) : n in [0..10]];
(PARI) {a(n) = if( n<0, 0, polcoeff( 1 / eta(x + x * O(x^n)), n))};
(PARI) /* The Hardy-Ramanujan-Rademacher exact formula in PARI is as follows (this is no longer necessary since it is now built in to the numbpart command): */
Psi(n, q) = local(a, b, c); a=sqrt(2/3)*Pi/q; b=n-1/24; c=sqrt(b); (sqrt(q)/(2*sqrt(2)*b*Pi))*(a*cosh(a*c)-(sinh(a*c)/c))
L(n, q) = if(q==1, 1, sum(h=1, q-1, if(gcd(h, q)>1, 0, cos((g(h, q)-2*h*n)*Pi/q))))
g(h, q) = if(q<3, 0, sum(k=1, q-1, k*(frac(h*k/q)-1/2)))
part(n) = round(sum(q=1, max(5, 0.5*sqrt(n)), L(n, q)*Psi(n, q)))
/* Ralf Stephan, Nov 30 2002, fixed by Vaclav Kotesovec, Apr 09 2018 */
(PARI) {a(n) = numbpart(n)};
(PARI) {a(n) = if( n<0, 0, polcoeff( sum( k=1, sqrtint(n), x^k^2 / prod( i=1, k, 1 - x^i, 1 + x * O(x^n))^2, 1), n))};
(PARI) f(n)= my(v, i, k, s, t); v=vector(n, k, 0); v[n]=2; t=0; while(v[1]<n, i=2; while(v[i]==0, i++); v[i]--; s=sum(k=i, n, k*v[k]); while(i>1, i--; s+=i*(v[i]=(n-s)\i)); t++); t \\ Thomas Baruchel, Nov 07 2005
(PARI) a(n)=if(n<0, 0, polcoeff(exp(sum(k=1, n, x^k/(1-x^k)/k, x*O(x^n))), n)) \\ Joerg Arndt, Apr 16 2010
(MuPAD) combinat::partitions::count(i) $i=0..54 // Zerinvary Lajos, Apr 16 2007
(Sage) [number_of_partitions(n) for n in range(46)]  # Zerinvary Lajos, May 24 2009
(Sage)
@CachedFunction
def A000041(n):
    if n == 0: return 1
    S = 0; J = n-1; k = 2
    while 0 <= J:
        T = A000041(J)
        S = S+T if is_odd(k//2) else S-T
        J -= k if is_odd(k) else k//2
        k += 1
    return S
[A000041(n) for n in range(50)]  # Peter Luschny, Oct 13 2012
(Sage) # uses[EulerTransform from A166861]
a = BinaryRecurrenceSequence(1, 0)
b = EulerTransform(a)
print([b(n) for n in range(50)]) # Peter Luschny, Nov 11 2020
(Haskell)
import Data.MemoCombinators (memo2, integral)
a000041 n = a000041_list !! n
a000041_list = map (p' 1) [0..] where
   p' = memo2 integral integral p
   p _ 0 = 1
   p k m = if m < k then 0 else p' k (m - k) + p' (k + 1) m
-- Reinhard Zumkeller, Nov 03 2015, Nov 04 2013
(Maxima) num_partitions(60, list); /* Emanuele Munarini, Feb 24 2014 */
(GAP) List([1..10], n->Size(OrbitsDomain(SymmetricGroup(IsPermGroup, n), SymmetricGroup(IsPermGroup, n), \^))); # Attila Egri-Nagy, Aug 15 2014
(Perl) use ntheory ":all"; my @p = map { partitions($_) } 0..100; say "[@p]"; # Dana Jacobsen, Sep 06 2015
(Racket)
#lang racket
; SUM(k, -inf, +inf) (-1)^k p(n-k(3k-1)/2)
; For k outside the range (1-(sqrt(1-24n))/6 to (1+sqrt(1-24n))/6) argument n-k(3k-1)/2 < 0.
; Therefore the loops below are finite. The hash avoids repeated identical computations.
(define (p n) ; Nr of partitions of n.
(hash-ref h n
  (λ ()
   (define r
    (+
     (let loop ((k 1) (n (sub1 n)) (s 0))
      (if (< n 0) s
       (loop (add1 k) (- n (* 3 k) 1) (if (odd? k) (+ s (p n)) (- s (p n))))))
     (let loop ((k -1) (n (- n 2)) (s 0))
      (if (< n 0) s
       (loop (sub1 k) (+ n (* 3 k) -2) (if (odd? k) (+ s (p n)) (- s (p n))))))))
   (hash-set! h n r)
   r)))
(define h (make-hash '((0 . 1))))
; (for ((k (in-range 0 50))) (printf "~s, " (p k))) runs in a moment.
; Jos Koot, Jun 01 2016
(Python)
from sympy.ntheory import npartitions
print([npartitions(i) for i in range(101)])  # Indranil Ghosh, Mar 17 2017
(Julia) # DedekindEta is defined in A000594
A000041List(len) = DedekindEta(len, -1)
A000041List(50) |> println # Peter Luschny, Mar 09 2018
```

CROSSREFS

Cf. A000009, A000079, A000203, A001318, A008284, A065446, A078506, A113685, A132311, A145006, A145007, A147843, A152537, A168532, A173238, A173239, A173241, A173304, A174065, A174066, A174068, A176202.
For successive differences see A002865, A053445, A072380, A081094, A081095.
Antidiagonal sums of triangle A092905. a(n) = A054225(n,0).
Boustrophedon transforms: A000733, A000751.
Cf. A167376 (complement), A061260 (multisets).

KEYWORD

core,easy,nonn,nice

AUTHOR

N. J. A. Sloane

EXTENSIONS Additional comments from Ola Veshta (olaveshta(AT)my-deja.com), Feb 28 2001
Additional comments from Dan Fux (dan.fux(AT)OpenGaia.com or danfux(AT)OpenGaia.com), Apr 07 2001

STATUS approved

A027750 Triangle read by rows in which row n lists the divisors of n.

1, 1, 2, 1, 3, 1, 2, 4, 1, 5, 1, 2, 3, 6, 1, 7, 1, 2, 4, 8, 1, 3, 9, 1, 2, 5, 10, 1, 11, 1, 2, 3, 4, 6, 12, 1, 13, 1, 2, 7, 14, 1, 3, 5, 15, 1, 2, 4, 8, 16, 1, 17, 1, 2, 3, 6, 9, 18, 1, 19, 1, 2, 4, 5, 10, 20, 1, 3, 7, 21, 1, 2, 11, 22, 1, 23, 1, 2, 3, 4, 6, 8, 12, 24, 1, 5, 25, 1, 2, 13, 26, 1, 3, 9, 27, 1, 2, 4, 7, 14, 28, 1, 29 (list; graph; refs; listen; history; edit; text; internal format)

OFFSET 1,3

COMMENTS
Or, in the list of natural numbers (A000027), replace n with its divisors.
This gives the first elements of the ordered pairs (a,b) a >= 1, b >= 1 ordered by their product ab.
Also, row n lists the largest parts of the partitions of n whose parts are not distinct. - Omar E. Pol, Sep 17 2008
Concatenation of n-th row gives A037278(n). - Reinhard Zumkeller, Aug 07 2011
{A210208(n,k): k=1..A073093(n)} subset of {T(n,k): k=1..A000005(n)} for all n. - Reinhard Zumkeller, Mar 18 2012
Row sums give A000203. Right border gives A000027. - Omar E. Pol, Jul 29 2012
Indices of records are in A006218. - Irina Gerasimova, Feb 27 2013
The number of primes in the n-th row is omega(n) = A001221(n). - Michel Marcus, Oct 21 2015
The row polynomials P(n,x) = Sum_{k=1..A000005(n)} T(n,k)*x^k with composite n which are irreducible over the integers are given in A292226. - Wolfdieter Lang, Nov 09 2017
T(n,k) is also the number of parts in the k-th partition of n into equal parts (see example). - Omar E. Pol, Nov 20 2019

LINKS
Franklin T. Adams-Watters, Rows 1..1000, flattened
Franklin T. Adams-Watters, Rows 1..10000
Omar E. Pol, Illustration of initial terms, (2009).
Eric Weisstein's World of Mathematics, Divisor
Wikipedia, Table of divisors
Index entries for sequences related to divisors of numbers

FORMULA
a(A006218(n-1) + k) = k-divisor of n, 1 <= k <= A000005(n). - Reinhard Zumkeller, May 10 2006
T(n,k) = n / A056538(n,k) = A056538(n,n-k+1), 1 <= k <= A000005(n). - Reinhard Zumkeller, Sep 28 2014

EXAMPLE
Triangle begins:
1;
1, 2;
1, 3;
1, 2, 4;
1, 5;
1, 2, 3, 6;
1, 7;
1, 2, 4, 8;
1, 3, 9;
1, 2, 5, 10;
1, 11;
1, 2, 3, 4, 6, 12;
...
For n = 6 the partitions of 6 into equal parts are [6], [3,3], [2,2,2], [1,1,1,1,1,1], so the number of parts are [1, 2, 3, 6] respectively, the same as the divisors of 6. - Omar E. Pol, Nov 20 2019

MAPLE
seq(op(numtheory:-divisors(a)), a = 1 .. 20) # **_Matt C. Anderson_**, May 15 2017

MATHEMATICA
Flatten[Table[Flatten [Divisors[n]], {n, 1, 30}]]

PROG
```
(MAGMA) [Divisors(n) : n in [1..20]];
(Haskell)
a027750 n k = a027750_row n !! (k-1)
a027750_row n = filter ((== 0) . (mod n)) [1..n]
a027750_tabf = map a027750_row [1..]
-- Reinhard Zumkeller, Jan 15 2011, Oct 21 2010
(PARI) v=List(); for(n=1, 20, fordiv(n, d, listput(v, d))); Vec(v) \\ Charles R Greathouse IV, Apr 28 2011
(Python)
from sympy import divisors
for n in range(1, 16):
    print(divisors(n)) # Indranil Ghosh, Mar 30 2017
```

CROSSREFS
Cf. A000005 (row length), A001221, A027749, A027751, A056534, A056538, A127093, A135010, A161700, A163280, A240698 (partial sums of rows), A240694 (partial products of rows), A247795 (parities), A292226, A244051.

KEYWORD nonn,easy,tabf,look

AUTHOR N. J. A. Sloane

EXTENSIONS More terms from Scott Lindhurst (ScottL(AT)alumni.princeton.edu)

STATUS approved

A016789 a(n) = 3*n + 2.

2, 5, 8, 11, 14, 17, 20, 23, 26, 29, 32, 35, 38, 41, 44, 47, 50, 53, 56, 59, 62, 65, 68, 71, 74, 77, 80, 83, 86, 89, 92, 95, 98, 101, 104, 107, 110, 113, 116, 119, 122, 125, 128, 131, 134, 137, 140, 143, 146, 149, 152, 155, 158, 161, 164, 167, 170, 173, 176, 179 (list; graph; refs; listen; history; edit; text; internal format)

OFFSET 0,1

COMMENTS
Except for 1, n such that Sum_{k=1..n} (k mod 3)*binomial(n,k) is a power of 2. - Benoit Cloitre, Oct 17 2002
The sequence 0,0,2,0,0,5,0,0,8,... has a(n) = n*(1 + cos(2*Pi*n/3 + Pi/3) - sqrt(3)*sin(2*Pi*n + Pi/3))/3 and o.g.f. x^2(2+x^3)/(1-x^3)^2. - Paul Barry, Jan 28 2004 [Artur Jasinski, Dec 11 2007, remarks that this should read (3*n + 2)*(1 + cos(2*Pi*(3*n + 2)/3 + Pi/3) - sqrt(3)*sin(2*Pi*(3*n + 2)/3 + Pi/3))/3.]
Except for 2, exponents e such that x^e + x + 1 is reducible. - N. J. A. Sloane, Jul 19 2005
a(n) = A125199(n+1,1). - Reinhard Zumkeller, Nov 24 2006
The trajectory of these numbers under iteration of sum of cubes of digits eventually turns out to be 371 or 407 (47 is the first of the second kind). - Avik Roy (avik_3.1416(AT)yahoo.co.in), Jan 19 2009
Union of A165334 and A165335. - Reinhard Zumkeller, Sep 17 2009
a(n) is the set of numbers congruent to{2,5,8} mod 9. - Gary Detlefs, Mar 07 2010
It appears that a(n) is the set of all values of y such that y^3 = kn + 2 for integer k. - Gary Detlefs, Mar 08 2010
These numbers do not occur in A000217 (triangular numbers). - Arkadiusz Wesolowski, Jan 08 2012
A089911(2*a(n)) = 9. - Reinhard Zumkeller, Jul 05 2013
Also indices of even Bell numbers (A000110). - Enrique Pérez Herrero, Sep 10 2013
Central terms of the triangle A108872. - Reinhard Zumkeller, Oct 01 2014
A092942(a(n)) = 1 for n > 0. - Reinhard Zumkeller, Dec 13 2014
a(n-1), n >= 1, is also the complex dimension of the manifold E(S), the set of all second order irreducible Fuchsian differential equations defined on P^1 = **C** U {oo}, having singular points at most in S = {a_1, ..., a_n, a_{n+1} = oo}, a subset of P^1. See the Iwasaki et al. reference, Proposition 2.1.3., p. 149. - Wolfdieter Lang, Apr 22 2016
Except for 2, exponents for which 1 + x^(n-1) + x^n is reducible. - Ron Knott, Sep 16 2016
The reciprocal sum of 8 distinct items from this sequence can be made equal to 1, with these terms: 2, 5, 8, 14, 20, 35, 41, 1640. - Jinyuan Wang, Nov 16 2018

There are no positive integers x, y, z such that 1/a(x) = 1/a(y) + 1/a(z). - Jinyuan Wang, Dec 31 2018
As a set of positive integers, it is the set sum S + S where S is the set of numbers in A016777. - Michael Somos, May 27 2019

REFERENCES

K. Iwasaki, H. Kimura, S. Shimomura and M. Yoshida, From Gauss to Painlevé, Vieweg, 1991. p. 149.
L. B. W. Jolley, "Summation of Series", Dover Publications, 1961, p. 16.
Konrad Knopp, Theory and Application of Infinite Series, Dover, p. 269

LINKS

G. C. Greubel, Table of n, a(n) for n = 0..10000
L. Euler, Observatio de summis divisorum p. 9.
L. Euler, An observation on the sums of divisors, arXiv:math/0411587 [math.HO], 2004-2009, p. 9.
INRIA Algorithms Project, Encyclopedia of Combinatorial Structures 937
Tanya Khovanova, Recursive Sequences
Konrad Knopp, Theorie und Anwendung der unendlichen Reihen, Berlin, J. Springer, 1922. (Original German edition of "Theory and Application of Infinite Series")
Luis Manuel Rivera, Integer sequences and k-commuting permutations, arXiv preprint arXiv:1406.3081 [math.CO], 2014-2015.
Index entries for linear recurrences with constant coefficients, signature (2,-1).

FORMULA

G.f.: (2+x)/(1-x)^2.
a(n) = 3 + a(n-1).
a(n) = 1 + A016777(n).
a(n) = A124388(n)/9.
Sum_{n>=1} (-1)^n/a(n) = (1/3)*(Pi/sqrt(3) - log(2)). - Benoit Cloitre, Apr 05 2002
1/2 - 1/5 + 1/8 - 1/11 + ... = (1/3)*(Pi/sqrt(3) - log 2). [Jolley] - Gary W. Adamson, Dec 16 2006
Sum_{n>=0} 1/(a(2*n)*a(2*n+1)) = (Pi/sqrt(3) - log 2)/9 = 0.12451569... (see A196548). [Jolley p. 48 eq (263)]
a(n) = 2*a(n-1) - a(n-2); a(0)=2, a(1)=5. - Philippe Deléham, Nov 03 2008
a(n) = 6*n - a(n-1) + 1 with a(0)=2. - Vincenzo Librandi, Aug 25 2010
a(n) = n XOR A005351(n+1) XOR A005352(n+1) (conjectured). - _Gilian Breysens_, Jul 21 2017
E.g.f.: (2 + 3*x)*exp(x). - _G. **C.** Greubel_, Nov 02 2018
a(n) = A005449(n+1) - A005449(n). - Jinyuan Wang, Feb 03 2019
a(n) = -A016777(-1-n) for all n in Z. - Michael Somos, May 27 2019

EXAMPLE

G.f. = 2 + 5*x + 8*x^2 + 11*x^3 + 14*x^4 + 17*x^5 + 20*x^6 + ... - Michael Somos, May 27 2019

MAPLE

```
seq(3*n+2, n = 0 .. 50); # _Matt C. Anderson_, May 18 2017
```

MATHEMATICA

```
Range[2, 500, 3] (* Vladimir Joseph Stephan Orlovsky, May 26 2011 *)
LinearRecurrence[{2, -1}, {2, 5}, 70] (* Harvey P. Dale, Aug 11 2021 *)
```

PROG

```
(Haskell)
a016789 = (+ 2) . (* 3)  -- Reinhard Zumkeller, Jul 05 2013
(PARI) vector(100, n, 3*n-1) \\ Derek Orr, Apr 13 2015
(MAGMA) [3*n+2: n in [0..80]]; // Vincenzo Librandi, Apr 14 2015
(GAP) List([0..70], n->3*n+2); # Muniru A Asiru, Nov 02 2018
(Python) for n in range(0, 100): print(3*n+2, end=', ') # Stefano Spezia, Nov 21 2018
```

CROSSREFS

First differences of A005449.
Cf. A002939, A017041, A017485, A125202, A017233, A179896, A017617, A016957, A008544 (partial products), A032766, A016777, A124388, A005351.
Cf. A087370.
Cf. similar sequences with closed form (2*k-1)*n+k listed in A269044.

KEYWORD

nonn,easy,changed

AUTHOR

N. J. A. Sloane

STATUS

approved

A005846 Primes of the form n^2 + n + 41.
(Formerly M5273)

41, 43, 47, 53, 61, 71, 83, 97, 113, 131, 151, 173, 197, 223, 251, 281, 313, 347, 383, 421, 461, 503, 547, 593, 641, 691, 743, 797, 853, 911, 971, 1033, 1097, 1163, 1231, 130

1373, 1447, 1523, 1601, 1847, 1933, 2111, 2203, 2297, 2393, 2591, 2693, 2797 (list; graph; refs; listen; history; edit; text; internal format)

OFFSET

1,1

COMMENTS

Note that 41 is the largest of Euler's Lucky numbers (A014556). - Lekraj Beedassy, Apr 22 2004
a(n) = A117530(13, n) for n <= 13: a(1) = A117530(13, 1) = A014556(6) = 41, A117531(13) = 13. - Reinhard Zumkeller, Mar 26 2006
The link to E. Wegrzynowski contains the following incorrect statement: "It is possible to find a polynomial of the form n^2 + n + B that gives prime numbers for n = 0, ..., A, A being any number." It is known that the maximum is A = 39 for B = 41. - Luis Rodriguez (luiroto(AT)yahoo.com), Jun 22 2008
Contrary to the last comment, Mollin's Theorem 2.1 shows that any A is possible if the Prime k-tuples Conjecture is assumed. - T. D. Noe, Aug 31 2009
a(n) can be generated by a recurrence based on the gcd in the type of Eric Rowland and Aldrich Stevens. See the recurrence in PARI under PROG. - Mike Winkler, Oct 02 2013
These primes are not prime in O_(Q(sqrt(-163)). Given p = n^2 + n + 41, we have ((2n + 1)/2 - sqrt(-163)/2)((2n + 1)/2 + sqrt(-163)/2) = p, e.g., 1601 = 39^2 + 39 + 41 = (79/2 - sqrt(-163)/2)(79/2 + sqrt(-163)/2). - Alonso del Arte, Nov 03 2017
From Peter Bala, Apr 15 2018: (Start)
The polynomial P(n) := n^2 + n + 41 takes distinct prime values for the 40 consecutive integers n = 0 to 39. It follows that the polynomial P(n-40) takes prime values for the 80 consecutive integers n = 0 to 79, consisting of the 40 primes above each taken twice. We note two consequences of this fact.
1) The polynomial P(2*n-40) = 4*n^2 - 158*n + 1601 also takes prime values for the 40 consecutive integers n = 0 to 39.
2) The polynomial P(3*n-40) = 9*n^2 - 237*n + 1601 takes prime values for the 27 consecutive integers n = 0 to 26 (= floor(79/3)). In addition, calculation shows that P(3*n-40) also takes prime values for n from -13 to -1. Equivalently put, the polynomial P(3*n-79) = 9*n^2 - 471*n + 6203 takes prime values for the 40 consecutive integers n = 0 to 39. This result is due to Higgins. Cf. A007635 and A048059. (End)

REFERENCES

O. Higgins, Another long string of primes, J. Rec. Math., 14 (1981/2) 185.
Paulo Ribenboim, The Book of Prime Number Records. Springer-Verlag, NY, 2nd ed., 1989, p. 137.
N. J. A. Sloane and Simon Plouffe, The Encyclopedia of Integer Sequences, Academic Press, 1995 (includes this sequence).

LINKS

Zak Seidov, Table of n, a(n) for n = 1..10000.
Phil Carmody, Drag Racing Prime Numbers! - Vladimir Joseph Stephan Orlovsky, Jul 24 2011
Richard K. Guy, The strong law of small numbers. Amer. Math. Monthly 95 (1988), no. 8, 697-712. [Annotated scanned copy]
R. A. Mollin, Prime-producing quadratics, Amer. Math. Monthly 104 (1997), 529-544.
E. Wegrzynowski, Les formules simples qui donnent des nombres premiers en grande quantité
Eric Weisstein's World of Mathematics, Euler Prime
Eric Weisstein's World of Mathematics, Prime-Generating Polynomial

FORMULA

a(n) = A056561(n)^2 + A056561(n) + 41.

EXAMPLE

a(39) = 1601 = 39^2 + 39 + 41 is in the sequence because it is prime.
1681 = 40^2 + 40 + 41 is not in the sequence because 1681 = 41*41.

MAPLE

```
for y from 0 to 10 do
U := y^2+y+41;
if isprime(U) = true then print(U) end if ;
end do:
# _Matt C. Anderson_, Jan 04 2013
```

MATHEMATICA

```
Select[Table[n^2 + n + 41, {n, 0, 59}], PrimeQ] (* Alonso del Arte, Dec 08 2011 *)
```

PROG

```
(PARI) for(n=1, 1e3, if(isprime(k=n^2+n+41), print1(k", "))) \\ Charles R Greathouse IV, Jul 25 2011
(Haskell)
a005846 n = a005846_list !! (n-1)
a005846_list = filter ((== 1) . a010051) a202018_list
-- Reinhard Zumkeller, Dec 09 2011
(PARI) {k=2; n=1; for(x=1, 100000, f=x^2+x+41; g=x^2+3*x+43; a=gcd(f, g-k); if(a>1, k=k+2); if(a==x+2-k/2, print(n" "a); n++))} \\ Mike Winkler, Oct
```

02 2013
(GAP) Filtered(List([0..100], n->n^2+n+41), IsPrime); # Muniru A Asiru, Apr 22 2018
(MAGMA) [a: n in [0..55] | IsPrime(a) where a is n^2+n+ 41]; // Vincenzo Librandi, Apr 24 2018

CROSSREFS Cf. A048988, A007634, A056561, A002378, A007635.
Intersection of A000040 and A202018; A010051.
Cf. A048059.

KEYWORD nonn,easy

AUTHOR N. J. A. Sloane

EXTENSIONS More terms from Henry Bottomley, Jun 26 2000

STATUS approved

A007530 Prime quadruples: numbers k such that k, k+2, k+6, k+8 are all prime. (Formerly M3816)

5, 11, 101, 191, 821, 1481, 1871, 2081, 3251, 3461, 5651, 9431, 13001, 15641, 15731, 16061, 18041, 18911, 19421, 21011, 22271, 25301, 31721, 34841, 43781, 51341, 55331, 62981, 67211, 69491, 72221, 77261, 79691, 81041, 82721, 88811, 97841, 99131 (list; graph; refs; listen; history; edit; text; internal format)

OFFSET 1,1

COMMENTS Except for the first term, 5, all terms == 11 (mod 30). - Zak Seidov, Dec 04 2008
Some further values: For k = 1, ..., 10, a(k*10^3) = 11721791, 31210841, 54112601, 78984791, 106583831, 136466501, 165939791, 196512551, 230794301, 265201421. - M. F. Hasler, May 04 2009
k is the first prime of 2 consecutive twin prime pairs. - Daniel Forgues, Aug 01 2009
The prime quadruples of form p + (0, 2, 6, 8) have the quadruple congruence class (-1, +1, -1, +1) (mod 6). - Daniel Forgues, Aug 12 2009
s = (p+8)-(p) = 8 is the smallest s giving an admissible prime quadruple form, for which the only admissible form is p + (0, 2, 6, 8), since (0, 2, 6, 8) is the only form not covering all the congruence classes for any prime <= 4. Since s is smallest, these prime quadruples are prime constellations (or prime quadruplets), i.e., they contain consecutive primes. - Daniel Forgues, Aug 12 2009
Except for the first term, 5, all prime quadruples are of the form (15k-4, 15k-2, 15k+2, 15k+4), with k >= 1, and so are centered on 15k. - Daniel Forgues, Aug 12 2009
Solutions of the equation n'+(n+2)'+(n+6)'+(n+8)'=4, where n' is the arithmetic derivative of n'. - Paolo P. Lava, Nov 09 2012
Subsequence of A022004. - R. J. Mathar, Feb 10 2013
The quadruplets are listed in A136162. - M. F. Hasler, Apr 20 2013
Starting at a(2) and examining the first 50 terms, (a(n)+4)/15 is a prime in 8 cases and a semiprime in 21; the last 18 terms have 2 primes and 11 semiprimes. Do the number of semiprimes continue to occur greater than mere chance? - J. M. Bergot, Apr 27 2015

REFERENCES H. Rademacher, Lectures on Elementary Number Theory. Blaisdell, NY, 1964, p. 4.
N. J. A. Sloane and Simon Plouffe, The Encyclopedia of Integer Sequences, Academic Press, 1995 (includes this sequence).

LINKS Matt C. Anderson, Table of n, a(n) for n = 1..10000 (terms 1..1000 from T. D. Noe).
C. K. Caldwell, The Prime Glossary, prime quadruple
T. R. Nicely, Enumeration to 1.6e15 of the prime quadruplets
H. Riesel, Prime numbers and computer methods for factorization, Progress in Mathematics, Vol. 57, Birkhäuser, Boston, 1985, ISBN: 978-0-8176-8297-2, Chap. 4, see p. 65.
Eric Weisstein's World of Mathematics, Prime Quadruplet

FORMULA a(n) = 11 + 30*A014561(n-1) for n > 1. - M. F. Hasler, May 04 2009

EXAMPLE From M. F. Hasler, May 04 2009: (Start)
a(1)=5 is the start of the first prime quadruplet, {5,7,11,13}.
a(2)=11 is the start of the second prime quadruplet, {11,13,17,19}, and all other prime quadruplets differ from this one by a multiple of 30.
a(100)=470081 is the start of the 100th prime quadruplet;
a(500)=4370081 is the start of the 500th prime quadruplet.
a(167)=1002341 is the least quadruplet prime beyond 10^6. (End)

MAPLE
```
A007530:=proc(q)
local n;
for n from 1 to q do
  if isprime(n) and isprime (n+2) and isprime(n+6) and isprime (n+8) then print(n); fi;
od; end:
A007530(10000000000);  # Paolo P. Lava, Jan 30 2013
```

MATHEMATICA
```
A007530 = Select[Range[1, 10^5 - 1, 2], Union[PrimeQ[# + {0, 2, 6, 8}]] == {True} &] (* Alonso del Arte, Sep 24 2011 *)
Select[Prime[Range[10000]], AllTrue[#+{2, 6, 8}, PrimeQ]&] (* The program uses the AllTrue function from Mathematica version 10 *) (* Harvey P. Dale, Mar 11 2019 *)
```

PROG
```
(PARI) A007530( n, print_all=0, s=2 )={ my(p, q, r); until(!n--, until( p+8==s=nextprime(s+2), p=q; q=r; r=s); print_all && print1(p", ")); p} \\
    The optional 3rd argument can be used to obtain large values by starting from some precomputed point instead of zero, using a(n+k)
    = A007530(k+1, , a(n)) (or A007530(k, , a(n)-1) for k>0); e.g., you get a(10^4+k) using A007530(k+1, , 265201421) (value of a(10^4) from the
    comments section). - M. F. Hasler, May 04 2009
(PARI) forprime(p=2, 10^5, if(isprime(p+6) && isprime(p+8), print1(p, ", "))) \\ Felix Fröhlich, Jun 22 2014
(MAGMA) [ p: p in PrimesUpTo(11000)| IsPrime(p+2) and IsPrime(p+6) and IsPrime(p+8)] // Vincenzo Librandi, Nov 18 2010
(Python)
from sympy import primerange
def aupto(limit):
  p, q, r, alst = 2, 3, 5, []
  for s in primerange(7, limit+9):
    if p+2 == q and p+6 == r and p+8 == s: alst.append(p)
    p, q, r = q, r, s
  return alst
print(aupto(10**5)) # Michael S. Branicky, May 11 2021
```

CROSSREFS Cf. A159910 (first differences divided by 30), A120120, A007811, A014561.

KEYWORD nonn

AUTHOR N. J. A. Sloane, Robert G. Wilson v

EXTENSIONS More terms from Warut Roonguthai
Incorrect formula and Mathematica program removed by N. J. A. Sloane, Dec 04 2008, at the suggestion of Zak Seidov
Values up to a(1000) checked with the given PARI code by M. F. Hasler, May 04 2009

STATUS approved

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